



## **Disentangling multi-level systems: averaging, correlations and memory**

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We consider two weakly coupled systems and adopt a perturbative approach based on the Ruelle response theory to study their interaction. We propose a systematic way to parametrize the effect of the coupling as a function of only the variables of a system of interest. Our focus is on describing the impacts of the coupling on the long-term statistics rather than on the finite-time behaviour. By direct calculation, we find that, at first order, the coupling can be surrogated by adding a deterministic perturbation to the autonomous dynamics of the system of interest. At second order, there are additionally two separate and very different contributions. One is a term taking into account the second order contributions of the fluctuations in the coupling, which can be parametrized as a stochastic forcing with given spectral properties. The other one is a memory term, coupling the system of interest to its previous history, through the correlations of the second system. If these correlations are known, this effect can be implemented as a perturbation with memory on the single system. In order to treat this case, we present an extension to Ruelle's response theory able to deal with integral operators. We discuss our results in the context of other methods previously proposed to disentangle the dynamics of two coupled systems. We emphasize that our results do not rely on assuming a time scale separation, and, if such a separation exist, can be used equally well to study the statistics of the slow as well as that of the fast variables. By recursively applying the technique proposed here, we can treat the general case of multilevel systems.