



Reproducing kernel for the exterior of an ellipsoid and its use for generating function bases in gravity field studies

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In gravity field studies very often linear combinations of basis functions are used to approximate the gravitational potential of the Earth or its disturbing part. The problem is frequently interpreted for the exterior of a sphere or an oblate ellipsoid of revolution. As a rule, spherical or ellipsoidal harmonics are used as basis functions within this concept. The second case is less frequent, but has a number of driving impulses and in general its investigation and possibilities for more routine implementation are given a considerable attention. As known basis functions like spherical or ellipsoidal harmonics are frequency localized. The purpose of this paper is to discuss an alternative way suitable for approximating harmonic functions in the exterior of the oblate ellipsoid of revolution. Our aim is to study the use of space localized basis functions. We focus on basis functions generated by means of the reproducing kernel in the respective Hilbert space. The use of the reproducing kernel offers a very straightforward way leading to entries in Galerkin's matrix of the respective linear system for unknown scalar coefficients. Note that for spherical case, apart from some algebra, technical problems associated with constructing the reproducing kernel and its use may be solved relatively easily. We will show it as an example. Nevertheless our effort aims to approximations in the exterior of the oblate ellipsoid of revolution. We show how the reproducing kernel can be obtained and give its series representation. The fundamental problem, however, is the possibility of practical summation of the series that represents the kernel. Indeed, it is difficult to reduce the number of summation indices since in the ellipsoidal case there is not a straightforward analogue to the addition theorem known for spherical situation. This, in consequence, makes the computation of the kernel and especially the set of the entries in Galerkin's matrix, even by means of high performance facilities, rather demanding. This is also the main problem discussed in the paper. The reproducing kernel is analyzed, split into parts and various methods and tools, including hypergeometric functions and series are used to represent its structure, so as to enable an effective numerical treatment of the kernel.