



## **A Novel Analytical Solution for Coupled Multi-Species Contaminant Transport in Finite Spatial Domain Subject to Arbitrary Time-Dependent Inlet Boundary Condition**

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Several analytical solutions for single-species reactive solute transport problems have been reported in literature for predicting the transport of various contaminants. Analytical solutions for coupled multi-species reactive solute transport problem are much more difficult and relatively rare in subsurface hydrology. Problem of coupled multi-species reactive transport plays an important role in understanding the transport and fate of a variety of decay chain contaminants such as radionuclide, chlorinated solvents, and nitrogen. Analytical solutions are efficient tools for testing and validating more comprehensively numerical models, performing sensitivity analyses to investigate how various transport processes affect contaminant transport, or serving as screening models. Decomposition strategy such as linear transform format or matrix diagonalization method which decomposes the set of coupled advective-dispersive transport equations into a system of independent differential equations have been widely used to derive the analytical solution for coupled multi-species solute transport problem. These decomposition approaches are mostly performed on the partial differential equations or ordinary differential equations. Generally, the processes of applying decomposition technique on differential equations are much more difficult, thus these solution methods are mostly limited to derive the analytical solution for either a semi-infinite spatial domain or steady-state boundary condition.

In this study we present a novel analytical solution to multi-species advective-dispersive transport equations sequentially coupled by first-order decay reactions in a finite spatial domain subject to arbitrary time-dependent inlet boundary condition. The novel solution is derived by consecutive applications of Laplace transform and the generalized integral transform to remove the temporal and spatial derivatives in a set of coupled advection-dispersion equations, thus converting the coupled partial differential equation system into a set of algebraic equations. Subsequently, simple mathematical manipulation is applied to solve the set of algebraic equations in transform domain and the analytical solution in the transformed domain for each species is independently obtained. Finally, the solutions for all species in transformed domain are transformed back into the original domain by successively executing Laplace and the generalized integral transform inversions. The developed analytical solutions for a finite spatial domain are compared with the analytical solution for a semi-finite spatial domain to investigate the impact of the exit boundary on coupled multi-species transport. The proposed solution method in this study has the greater flexibility in dealing with analytical model for more complicated problems, thus will be especially useful for expanding the number and type of analytical models for sequentially coupled multi-species reactive transport problem.