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Orthogonality of Harmonic Potentials and Fields in Spheroidal Coordinates (Petrus Peregrinus Medal Lecture)

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Spherical harmonic scalar potentials are the independent solutions of the Laplace equation relevant in a spherical geometry; they are used widely in global geomagnetism and geodesy to represent the situation in the source-free region outside the Earth. It is well known that these harmonics are orthogonal over the sphere, as are the vector fields that are the gradients of the harmonics. If we have data (potential or field) over the sphere, this orthogonality enables us to use spherical harmonic analysis to determine separately the numerical coefficient relevant for each harmonic potential.
But the Earth is better approximated by an oblate spheroid; and for sources near the surface it is more relevant to use a spheroidal coordinate system; in this case the appropriate solutions of the Laplace equation are now spheroidal harmonics. However these SPHEROIDAL harmonics are NOT orthogonal over the SPHEROID, and neither are the corresponding vector potential gradients. I show how this problem can be overcome by using an appropriate weighting factor that depends only on colatitude; the factor is different for potential and for field. By using the appropriate weighting factor it is then possible to do the spheroidal equivalent of spherical harmonic analysis, either for the scalar potential or the corresponding vector field.
In the spherical case, because of the orthogonality it is possible to separate the total mean-square potential over the sphere into parts contributed by harmonics of different degrees, e.g. the 'degree variance' in geodesy. Similarly, in geomagnetism we have the 'power spectrum' that separates the total mean-square vector field into contributions from different degrees. But in the spheroidal case such a separation (of potential or field) is possible only if we use a WEIGHTED mean-square.