



Theoretical Study of Wave Breaking for Nonlinear Water Waves Propagating on a Sloping Bottom

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In this paper, a third-order asymptotic solution in a Lagrangian framework describing nonlinear water wave propagation on the surface of a uniform sloping bottom is presented. A two-parameter perturbation method is used to develop a new mathematical derivation. The particle trajectories, wave pressure and Lagrangian velocity potential are obtained as a function of the nonlinear wave steepness and the bottom slope perturbed to third order. This theoretical solution in Lagrangian form satisfies state of the normal pressure at the free surface. The condition of the conservation of mass flux is examined in detail for the first time. The two important properties in Lagrangian coordinates, Lagrangian wave frequency and Lagrangian mean level, are included in the third-order solution. The solution can also be used to estimate the mean return current for waves progressing over the sloping bottom. The Lagrangian solution untangle the description of the features of wave shoaling in the direction of wave propagation from deep to shallow water, as well as the process of successive deformation of a wave profile and water particle trajectories leading to wave breaking. A series of experiment was conducted to validate the obtained theoretical solution.

The proposed solution will be used to determine the wave shoaling and breaking process and the comparisons between the experimental and theoretical results are excellent. For example, the variations of phase velocity on sloping bottom are obtained by 7 set of two close wave gauges and the theoretical result could accurately predict the measured phase velocity. The theoretical wave breaking index can be derived by use of the kinematic stability parameter (K.P.S). The comparisons between the theory, experiment (present study, Iwagali et al.(1974), Deo et al.(2003) and Tsai et al.(2005)) and empirical formula of Goda (2004) for the breaking index(u/C) versus the relative water depth(d/L) under two different bottom slopes shows that the theoretical breaking index is well agreement with the experimental results for three bottom slopes. However, for steep slope of $1/3$, the result of Goda's empirical formula gives a larger deviation in comparison with the experimental data and the present theory. Some of empirical formulas presented the breaking wave height in terms of deepwater wave condition, such as in Sunamura (1983) and in Rattanapitikon and Shibayama(2000). Finally, the theoretical results are in good agreement with the experimental data The empirical formula of Sunamura (1983) always predicts an overestimated value and the formula of Rattanapitikon & Shibayama(2000) underestimate the breaking wave height for the steep bottom slope.