



## Application of a normalized Nash-Sutcliffe efficiency to improve the accuracy of the Sobol' sensitivity analysis of a hydrological model

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Sensitivity analysis (SA) has become a main practice in hydrological modelling, since it allows to identify the influential and non-influential parameters of a model and can give insights on the model processes and their relation with the system. A very robust SA technique that is becoming popular in hydrology is the Sobol' method (Sobol', 1990). This method quantifies the amount of variance that each parameter contributes to the unconditional variance of the model output. This variance contribution is expressed with sensitivity indices, which are assessed by means of Monte Carlo integrals. Hereto, a large number of random points are sampled in the parameter hyperspace to evaluate the model.

When the Sobol' method is applied to assess the influence of the model parameters on simulated time series, an objective function is required to transform the vector output of the model into a scalar input for the SA. Since the accuracy of the variance estimation with the numerical integrals may decrease when the mean value of the scalar inputs for the SA is large (Sobol', 2001), the Nash-Sutcliffe efficiency (NSE) is assumed to yield more accurate results than e.g. the also commonly used Sum of Squared Residuals (SSR). In our application on a SWAT model of the Kleine Nete catchment (Belgium) (Nossent et al., 2011), this is indeed valid for flow predictions, as the mean NSE for all model evaluations is -0.73. However, for water quality simulations with the same model, the mean NSE values become highly negative (even an extreme value of -4E6 is obtained for nitrate concentration simulations). In such cases, the Nash-Sutcliffe efficiency is not suitable for the Sobol' sensitivity analysis.

We therefore introduce a normalized version of the Nash-Sutcliffe efficiency (NNSE) that yields values between 0 and 1, but preserves the main characteristics of the regular NSE:

$$NNSE = \frac{1}{2 - NSE} = \frac{\sum_i (o_i - \bar{o})^2}{\sum_i (o_i - s_i)^2 + (o_i - \bar{o})^2}$$

where  $s_i$  is the simulated value on day  $i$ ,  $o_i$  is the observed value on day  $i$  and  $\bar{o}$  is the average of the observations. As for the regular NSE, 1 is the optimal value for the NNSE. On the other hand, a value of 0.5 for the NNSE corresponds with a 0 value for the NSE, whereas the worst NNSE value is 0. As a consequence, the mean value of the scalar inputs for the SA is for the different variables in our SWAT model smaller than 0.5 and mostly even less than 0.05, which increases the accuracy of the variance estimates.

Besides the introduction of this normalized Nash-Sutcliffe efficiency, our presentation will furthermore provide evidence on the influence of the applied objective function on the outcome of the sensitivity analysis.

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