



## **A Unified Theory of Rainfall Extremes, Rainfall Excesses, and IDF Curves**

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Extreme rainfall events are a key component of hydrologic risk management and design. Yet, a consistent mathematical theory of such extremes remains elusive. This study aims at laying new statistical foundations for such a theory.

The quantities of interest are the distribution of the annual maximum, the distribution of the excess above a high threshold  $z$ , and the intensity-duration-frequency (IDF) curves. Traditionally, the modeling of annual maxima and excesses is based on extreme value (EV) and extreme excess (EE) theories. These theories establish that the maximum of  $n$  *iid* variables is attracted as  $n \rightarrow \infty$  to a generalized extreme value (GEV) distribution with a certain index  $k$  and the distribution of the excess is attracted as  $z \rightarrow \infty$  to a generalized Pareto distribution with the same index. The empirical value of  $k$  tends to decrease as the averaging duration  $d$  increases.

To a first approximation, the IDF intensities scale with  $d$  and the return period  $T$ . Explanations for this approximate scaling behavior and theoretical predictions of the scaling exponents have emerged over the past few years. This theoretical work has been largely independent of that on the annual maxima and the excesses. Deviations from exact scaling include a tendency of the IDF curves to converge as  $d$  and  $T$  increase.

To bring conceptual clarity and explain the above observations, we analyze the extremes of stationary multifractal measures, which provide good representations of rainfall within storms. These extremes follow from large deviation theory rather than EV/EE theory. A unified framework emerges that (a) encompasses annual maxima, excesses and IDF values without relying on EV or EE asymptotics, (b) predicts the index  $k$  and the IDF scaling exponents, (c) explains the dependence of  $k$  on  $d$  and the deviations from exact scaling of the IDF curves, and (d) explains why the empirical estimates of  $k$  tend to be positive (in the Frechet range) while, based on frequently assumed marginal distributions, EV/EE theory gives  $k = 0$ .