



Fourier-Series Expansion of Spherical Harmonic Functions

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In this study we investigate the Fourier series expansion of Legendre Functions (LF), the meridional function of spherical harmonics. Since the Legendre functions of even (odd) zonal wavenumber are represented with polynomials of cosine of colatitude (polynomials multiplied by sine of colatitude), they can be expressed as finite series of half-ranged cosine series (sine series, respectively). The Fourier coefficient of LF, which is defined as its projection to LF, was obtained using a stable recursion equation. The recursion equation consists of four terms with the coefficients less than or very close to unity, and needs initial conditions for the zonal mode, zonal wavenumber one, and sectorial modes. The initial conditions were given in explicit one-term formula for better accuracy. Computations of Fourier coefficients are carried out up to the degree and/or order 10 800 which is equivalent to one arcmin resolution. The errors of the Fourier coefficients in the double precision arithmetic, estimated in terms of orthogonality, normality, and comparison with reference values predicted by theory, are found to be in the order of machine round off. The new method is used for the calculation of power spectrum (or degree variance) of the global topography data of two, four, and ten arcmin resolution provided by National Geophysical Data Center of the National Oceanic and Atmospheric Administration. The spectrum showed a -2 slope over wide range, except for the degrees smaller than five (with almost flat spectrum) and the small-scale sub-range with the degree larger than 1 000 (of slope of $-7/2$). Fourier-series of LF is applied to the isotropic filtering of global-domain gridded data with equiangular distance. The accuracy and procedure involved in the filtering will be discussed in detail and compared with those obtained by other methods.