Geophysical Research Abstracts
Vol. 14, EGU2012-3553-1, 2012
EGU General Assembly 2012

Fisher information analysis in electrical impedance tomography

S. Nordebo (1), T. Sjödén (1), M. Gustafsson (2), and F. Soldovieri (3)

(1) Linnaeus University, School of Computer Science, Physics and Mathematics, Sweden (sven.nordebo@lnu.se), (2) Lund University, Department of Electrical and Information Technology, Sweden (mats.gustafsson@eit.lth.se), (3) National Research Council, Institute for Electromagnetic Sensing of the Environment (soldovieri.f@irea.cnr.it)

In this contribution it is demonstrated how the Cramér-Rao lower bound provides a quantitative analysis of the optimal accuracy and resolution in inverse imaging, see also Nordebo et al., 2010, 2010b, 2010c. The imaging problem is characterized by the forward operator and its Jacobian. The Fisher information operator is defined for a deterministic parameter in a real Hilbert space and a stochastic measurement in a finite-dimensional complex Hilbert space with Gaussian measure. The connection between the Fisher information and the Singular Value Decomposition (SVD) based on the Maximum Likelihood (ML) criterion (the ML-based SVD) is established. It is shown that the eigenspaces of the Fisher information provide a suitable basis to quantify the trade-off between the accuracy and the resolution of the (non-linear) inverse problem. It is also shown that the truncated ML-based pseudo-inverse is a suitable regularization strategy for a linearized problem, which exploits a sufficient statistics for estimation within these subspaces.

The statistical-based Cramér-Rao lower bound provides a complement to the deterministic upper bounds and the L-curve techniques that are employed with linearized inversion (Kirsch, 1996; Hansen, 1992, 1998, 2010). To this end, the Electrical Impedance Tomography (EIT) provides an interesting example where the eigenvalues of the SVD usually do not exhibit a very sharp cut-off, and a trade-off between the accuracy and the resolution may be of practical importance. A numerical study of EIT is described, including a statistical analysis of the model errors due to the linearization. The Fisher information and sensitivity analysis is also used to compare, evaluate, and optimize measurement configurations in EIT.

Acknowledgement

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under Grant Agreement no 225663.

References

[1] S. Nordebo, A. Fhager, M. Gustafsson, and B. Nilsson. A Green's function approach to Fisher information analysis and preconditioning in microwave tomography. *Inverse Problems in Science and Engineering*, 18(8):1043–1063, 2010.

[2] S. Nordebo, R. Bayford, B. Bengtsson, A. Fhager, M. Gustafsson, P. Hashemzadeh, B. Nilsson, T. Rylander, and T. Sjödén. An adjoint field approach to Fisher information-based sensitivity analysis in electrical impedance tomography. *Inverse Problems*, 26, 2010. 125008.

[3] S. Nordebo, M. Gustafsson, T. Sjödén, and F. Soldovieri. Data fusion for electromagnetic and electrical resistive tomography based on maximum likelihood. *International Journal of Geophysics*, pages 1–11, 2011. Article ID 617089.

[4] Andreas Kirsch. *An Introduction to the Mathematical Theory of Inverse Problems*. Springer-Verlag, New York, 1996.

[5] P. C. Hansen. Analysis of discrete ill-posed problems by means of the L-curve. *SIAM REVIEW*, 34(4):561–580, Dec 1992.

[6] P. C. Hansen. *Rank-Deficient and Discrete Ill-Posed Problems: Numerical Aspects of Linear Inversion*. SIAM-Society for Industrial and Applied Mathematics, 1998.

[7] P. C. Hansen. *Discrete Inverse Problems: Insight and Algorithms*. SIAM-Society for Industrial and Applied Mathematics, 2010.