



Combining different types of data for ill-posed geophysical/geodetic problems

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Regularization is crucial to solve geophysical and geodetic ill-posed inverse problems. In practice, we may combine different types of data to solve such an ill-posed inverse problem; the accuracy of these different types of data may not be known precisely and should be modelled by a number of unknown variance components. Although the weighting factors, or equivalently the variance components, can significantly affect joint inversion results of geophysical ill-posed problems, they have been either assumed to be known or empirically chosen. No solid statistical foundation is available yet to correctly determine the weighting factors of different types of data in joint geophysical inversion. In this case, all regularization techniques may not be proper to apply, unless techniques of variance component estimation are directly implemented to determine the correct weighting factors for each type of data. In this paper, we will solve ill-posed inverse problems by simultaneously determining the regularization parameter and the weighting factors of different types of data, either by using the criteria of mean squared errors or the cross validation. First we analyze the biases of estimated variance components due to the regularization parameter and then propose bias-corrected variance component estimators. We simulate two examples: a purely mathematical integral equation of the first kind modified from the first example of Phillips (1962) and a typical geophysical example of downward continuation to recover the gravity anomalies on the surface of the Earth from satellite measurements. Based on the two simulated examples, we extensively investigate the MSE and the iterative GCV methods. The simulated results have shown that these methods work well to correctly recover the unknown variance components and determine the regularization parameter. In other words, our methods let data speak for themselves, decide the correct weighting factors of different types of geophysical data, and determine the regularization parameter. In addition, we derive unbiased estimators of the noise variance by correcting the biases of the regularized residuals. The two new estimators of the noise variance are compared with six existing methods through numerical simulations. The simulation results have shown that the two new estimators perform as well as Wahba's estimator for highly ill-posed problems and outperform any existing methods for moderately ill-posed problems.