



Computing Streamfunction and Velocity Potential in a Limited Domain of Arbitrary Shape

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A new approach to solve for streamfunction and velocity potential in a limited domain of arbitrary shape is developed by minimizing the difference between the domain-integrated kinetic energy of the original horizontal velocity and that of the reconstructed one. The non-uniqueness of solution and compatibility between the coupled boundary conditions in computing velocity potential and streamfunction from horizontal velocity in a limited domain of arbitrary shape are revisited theoretically with rigorous mathematical treatments. Classic integral formulas and their variants are used to formulate solutions for the coupled problems.

When there is no inner boundary (around a data hole) inside the domain, the total solution is the sum of the internally and externally induced parts. For the internally induced part, three numerical schemes (grid-staggering, local-nesting and piecewise continuous integration) are designed to deal with the singularity of the Green's function encountered in numerical calculations. For the externally induced part, by setting the velocity potential (or streamfunction) component to zero, the other component of the solution can be computed in two ways: (1) Solve for the density function from its boundary integral equation and then construct the solution from the boundary integral of the density function. (2) Use the Cauchy integral to construct the solution directly. The boundary integral can be discretized on a uniform grid along the boundary. By using local-nesting (or piecewise continuous integration), the scheme is refined to enhance the discretization accuracy of the boundary integral around each corner point (or along the entire boundary).

When the domain is not free of data holes, the total solution contains a data-hole-induced part, and the Cauchy integral method is extended to construct the externally induced solution with irregular external and internal boundaries. An automated algorithm is designed to facilitate the integrations along the irregular external and internal boundaries. Numerical experiments are performed to evaluate the accuracy and efficiency of each scheme relative to others.

A practical test of this method to divide the divergent and rotational flows in a limited domain with terrains is performed with a comparison to results obtained in previous studies. The validation procedure shows the practical use of this wind separation scheme and its mathematical application in solving Poisson equations.