



## **1D Runoff-runon stochastic model in the light of queueing theory : heterogeneity and connectivity**

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Runoff production on a hillslope during a rainfall event may be simplified as follows. Given a soil of constant infiltrability  $I$ , which is the maximum amount of water that the soil can infiltrate, and a constant rainfall intensity  $R$ , runoff is observed where  $R$  is greater than  $I$ . The infiltration rate equals the infiltrability when runoff is produced,  $R$  otherwise. When ponding time, topography, and overall spatial and temporal variations of physical parameters, such as  $R$  and  $I$ , are neglected, the runoff equation remains simple.

In this study, we consider soils of spatially variable infiltrability. As runoff can re-infiltrate on down-slope areas of higher infiltrabilities (runon), the resulting process is highly non-linear. The stationary runoff equation is:

$$Q_{n+1} = \max(Q_n + (R - I_n) * \Delta x, 0)$$

where  $Q_n$  is the runoff arriving on pixel  $n$  of size  $\Delta x$  [ $L^2/T$ ],  $R$  and  $I_n$  the rainfall intensity and infiltrability on that same pixel [ $L/T$ ]. The non-linearity is due to the dependence of infiltration on  $R$  and  $Q_n$ , that is runon. This re-infiltration process generates patterns of runoff along the slope, patterns that organise and connect to each other differently depending on the rainfall intensity and the nature of the soil heterogeneity. The runoff connectivity, assessed using the connectivity function of Allard (1993), affects greatly the dynamics of the runoff hillslope.

Our aim is to assess, in a stochastic framework, the runoff organization on 1D slopes with random infiltrabilities (log-normal, exponential, bimodal and uniform distributions) by means of theoretical developments and numerical simulations. This means linking the nature of soil heterogeneity with the resulting runoff organisation. In term of connectivity, we investigate the relations between structural (infiltrability) and functional (runoff) connectivity.

A theoretical framework based on the queueing theory is developed. We implement the idea of Jones et al. (2009), who remarked that the above formulation is identical to the waiting time equation in a single server queue. Thanks to this theory, it is possible to accurately describe some outputs of our numerical model, notably the runoff repartition over the slope for uncorrelated exponential infiltrability distributions. Alternative formulations for the connectivity function of Allard (which cannot be predicted theoretically to our knowledge) are discussed with regard to predictability, efficiency in computation and qualification of the "near-connectedness" state of the system.