



Probability density function (Pdf) of daily rainfall depths by means of superstatistics of hydro-climatic fluctuations for African test cities

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The dynamic of hydro-climatic processes can fluctuate in a wide range of temporal scales. Such fluctuations are often unpredictable for ecosystems and the adaptation to these represent the great challenge for the survival and the stability of the species. An unsolved issue is how much these fluctuations of climatic variables to different temporal scales can influence the frequency and the intensity of the extreme events, and how much these events can modify the ecosystems life.

It is by now widespread that an increment of the frequency and the intensity of the extreme events will represent one of the strongest characteristic of the global climatic change, with the greatest social and biotics implications (Porporato et al 2006). Recent field experiments (Gutshick and BassiriRad, 2003) and numerical analysis (Porporato et al 2004) have shown that the extreme events can generate not negligible consequences on organisms of water-limited ecosystems. Adaptation measures and species and ecosystems answers to the hydro-climatic variations, is therefore strongly interconnected to the probabilistic structure of these fluctuations.

Generally the not-linear intermittent dynamic of a state variable z (a rainfall depth or the interarrival time between two storms), at short time scales (for example daily) is described by a probability density function (pdf), $p(z|v)$, where v is the parameter of the distribution. If the same parameter v varies so that the external forcing fluctuates at longer temporal scale, z reaches a new “local” equilibrium. When the temporal scale of the variation of v is larger than the one of z , the probability distribution of z can be obtained as a overlapping of the temporary equilibria (“*Superstatistic*” approach), i.e.:

$$p(z) = \int p(z|v) \cdot \varphi(v) dv \quad (1)$$

where $p(z|v)$ is the conditioned probability of z to v , while $\varphi(v)$ is the pdf of v (Beck, 2001; Benjamin and Cornell, 1970).

The present work, carried out within FP7-ENV-2010 CLUVA (CLimate Change and Urban Vulnerability in Africa) Project, shows the results relating to data processing of rainfall daily data with reference to the city of Ouagadougou (Burkina Faso) and Dar Es Salaam (Tanzania). In particular, with the help of “*Superstatistics*”, such processing allowed, taking account of the hydro-climatic fluctuations, to define a pdf for daily rainfall depths and interarrivals substantially better than common probability distributions present in literature.

In particular the pdf for the rainfall daily depths (h) is:

$$p(h) = \frac{1}{b_{1h} \cdot (2 + h^{3/2})} \quad (2)$$

in which, as Poisson exponential model, there is only a parameter b_{1h} . Considering the climatic fluctuations of b_{1h} , by means of inferential analysis the pdf for this parameter is the two parameters (α e β) Frechét distribution:

$$g(b_{1h}) = \frac{\alpha}{\beta} \cdot \left(\frac{\beta}{b_{1h}}\right)^{\alpha+1} \cdot \exp\left(-\left(\frac{\beta}{b_{1h}}\right)^{\alpha}\right) \quad (3)$$

and so by applying the (1) the h pdf is:

$$p(h) = \frac{1}{\beta \cdot (2 + h^{3/2})} \cdot \Gamma\left(\frac{\alpha + 1}{\alpha}\right) \quad (4)$$

where Γ is the gamma function.