



## Using shadowing ratios to evaluate data assimilation techniques

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Identifying successful "noise reduction" as such remains a challenge in applications where the "true" values are not known *a priori* (and linear noise reduction is a game not a true task). We suggest shadowing ratios as a measure of noise reduction when the task at hand involves prediction. Initial condition uncertainty will more or less always limit the lead time of a chaotic model, even when that model reproduces the system dynamics perfectly. Since in reality, observations from such systems tend to be clouded by measurement error, the maximum lead time we can expect to accurately predict using the model will be short. Data assimilation techniques attempt to improve our state estimates, we introduce a new measure which allows us to estimate the quality of these techniques. A model trajectory shadows for as long as it is consistent with the noise model of the observed states. We define the shadowing ratio as the ratio of the length of time the model shadows using the assimilated initial conditions to the length of time the model shadows using some reference data assimilation technique. We use the measure to evaluate the effectiveness of one assimilation technique in particular, Gradient Descent of Indeterminism (GDI). Using the Moore-Spiegel system as an example we first use shadowing ratios to show the effect of using different numbers of observations from the past when applying GDI. We then compare GDI to other assimilation techniques using the measure to compare the effectiveness from a forecasting perspective. Finally, since GDI requires derivative information from the system, we compare the effectiveness of the algorithm when using the exact derivative matrix and when approximating using a forward difference technique. Other aspects of GDI are discussed, in the context of filtering and noise reduction.