



Symmetries of Inhomogeneous Fluid Equations: from Balanced Relationships to the Internal Waves Equations

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The system of governing equations of an inhomogeneous fluid mechanics, including the basic environment (water or air) and stratified component (salt or steam) is based on balanced relationships for the medium with the thermodynamic equation of state far from the points of phase transitions that arise from the basic conservation laws of classical mechanics. The basic system of equations admits a finite-dimensional algebra generators of symmetry groups, which is a consequence of the homogeneity of space – time, and the isotropy of space (with the equality of inertial and gravitational masses), and the invariance of the equations of motion under Galilean transformations.

We investigated the influence of the main approximations used in oceanography on the general properties of models of fluid flows. In particular, the linearized equations of an incompressible stratified fluid lose their invariance with respect to Galilean transformations, its invariant properties are simplified. However, if we extend the class of admissible transformations, and write the equations in the space of Fourier images (equation of monochromatic internal waves), then governing equations will admit the Lorentz group of symmetry which acts in 2D space, where time plays the role of the vertical coordinate. Thus, the symmetries of the internal waves in Fourier space coincide with the symmetries of Maxwell's equations in empty space. The role of the time variable in this case will play the vertical coordinate, and the speed of light – the phase velocity of internal waves on the dimensionless frequency of oscillation. Then the basic properties of internal waves will be determined by the pseudo-Euclidean space.

Cause-related events will lie in a cone with an angle depending on ratio of the wave to buoyancy frequency describing “St. Andrew's cross” of internal waves. Derived expressions are the analogues of the Lorentz transformations of relativistic mechanics.

Thus, for group analysis of the fundamental equations of mechanics of inhomogeneous fluids, and its common approximations showed that the only fundamental model that is invariant relatively ten-parametric Galilean group. Simplifying assumptions substantially change the invariant properties of the governing systems of equations. This can include the extension and contraction of the symmetry group admitted that constitutes a violation of the conditions of equivalence of the original and derivative systems. Thus, the use of approximation of incompressibility and the transition to the Navier – Stokes equations and the boundary layer leads to expansion of the concept of inertial reference system for all systems moving relative to each other with an arbitrary linear acceleration. Along with the new symmetry properties of the approximate models arise and solutions that reflect the properties that did not have the original fundamental model.

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