



Higher-order weakly nonlinear theory for internal waves in three-layer fluid

O.E. Kurkina (1,2,3), A.A. Kurkin (2), and E.A. Rouvinskaya (1)

(1) Department of Mathematics, National Research University Higher School of Economics, Nizhny Novgorod, Russia (oksana.kurkina@mail.ru), (2) Department of Applied Mathematics, R.E. Alekseev Nizhny Novgorod State Technical University, Nizhny Novgorod, Russia, (3) Wave Engineering Laboratory, Institute of Cybernetics, Tallinn University of Technology, Tallinn, Estonia

Three-layer stratifications are proved to be a proper approximation of sea water density profile in some basins in the World Ocean with specific hydrological conditions. Some shallow basins such as the Baltic Sea and some river estuaries have more or less continuous three-layer vertical structure caused by the interplay of fresh water discharge to the surface and salt water intrusion in the bottom layers. In order to describe the basic features of the internal wave field in such environments it is necessary to introduce a three-layer model. Such models are considerably more complex than the most popular two-layer systems; however, they represent new dynamical effects and allow for much more analytical progress in their studies compared to the fully stratified situation.

In the present study two modes of long internal gravity waves in a three-layer fluid are investigated in the framework of higher-order nonlinear evolutionary equations derived with the use of asymptotic procedure from the governing Euler equations for inviscid incompressible layered medium with "rigid lid" and horizontal impermeable bottom. The equations are written upto the fifth order of the perturbation theory for both interfaces for the waves of both modes: first (fast mode) and second (slow mode, so-called double-humped or varicose). For each equation the coefficients of nonlinearity, dispersion and nonlinear dispersion are expressed explicitly in terms of parameters of this fluid configuration. The behavior and signs of the coefficients are analyzed. The necessary order of the equations is discussed and determined for each case. A few nonlinear asymptotic transformations are proposed to reduce higher-order equations to simpler lower-order or well-known integrable equations (Korteweg – de Vries, Gardner equations). Special attention is paid to the situations when the nonlinear terms of lower orders of perturbation theory can vanish. For such situations particular rescaling is performed in order to balance nonlinearity and dispersion terms in the equations. Solitary wave solutions are found for the derived equations and initial problem is considered for the combination of boxed impulses of different polarities, different regimes are marked out. The comparison to the theory of conjugate flows and to the existing observations is carried out. The asymmetry of the interfacial displacements for the lower and the upper interface is quantified for all the cases.

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