



Reduced Monte Carlo methods for the solution of stochastic groundwater flow problems

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Reduced order modeling is often employed to decrease the computational cost of numerical solutions of parametric Partial Differential Equations. Reduced basis, balanced truncation, projections methods are among the most studied techniques to achieve model reduction. We study the applicability of snapshot-based Proper Orthogonal Decomposition (POD) to Monte Carlo (MC) simulations applied to the solution of the stochastic groundwater flow problem. POD model reduction is obtained by projecting the model equations onto a space generated by a small number of basis functions (principal components). These are obtained upon exploring the solution (probability) space with snapshots, i.e. system states obtained by solving the original process-based equations. The reduced model is then employed to complete the ensemble by adding multiple realizations. We apply this technique to a two dimensional simulation of steady state saturated groundwater flow, and explore the sensitivity of the method to the number of snapshots and associated principal components in terms of accuracy and efficiency of the overall MC procedure. In our preliminary results, we distinguish the problem of heterogeneous recharge, in which the stochastic term is confined to the forcing function (additive stochasticity), from the case of heterogeneous hydraulic conductivity, in which the stochastic term is multiplicative. In the first scenario, the linearity of the problem is fully exploited and the POD approach yields accurate and efficient realizations, leading to substantial speed up of the MC method. The second scenario poses a significant challenge, as the adoption of a few snapshots based on the full model does not provide enough variability in the reduced order replicates, thus leading to poor convergence of the MC method. We find that increasing the number of snapshots improves the convergence of MC but only for large integral scales of the log-conductivity field. The technique is then extended to take full advantage of the solution of moment differential equations of groundwater flow.