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Hamiltonian discontinuous Galerkin FEM for linear inertial waves

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A compatible numerical scheme for a linear, three-dimensional, rotating (in)compressible fluid flow is presented. The scheme is based on discontinuous Galerkin FEM discretisation of corresponding Hamiltonian dynamics. The latter approach ensures a conservation of important mathematical properties of the partial differential equations (e.g. mass, energy, vorticity, phase space volume). Dirac theory is applied to derive the incompressible limit of the compressible Hamiltonian system which is used as a starting point in the numerical discretisation.
Due to the presence of Coriolis forces caused by the background rotation of the domain, the numerical scheme admits complicated wave solutions. The waves involved are so-called inertial waves, of relevance in oceanography and also for (filled) rotating fuel tanks. These inertial waves display multi-scale features with chaotic attractors in zones of intense wave activity. Thus, numerical algorithms and simulations of inertial waves are nontrivial.
The following challenges were encountered: (i) the discretisation of an incompressible flow or a divergence-free velocity field, a classical issue in computational fluid dynamics; (ii) discretisation of the special, geostrophic, boundary conditions combined with no-normal flow at solid walls; (iii) discretisation of the conserved, Hamiltonian dynamics of the inertial-waves; and, (iv) the large-scale computational demands owing to the inherently three-dimensional nature of inertial-wave dynamics and possibly its narrow zones of chaotic attraction.
Convergent and efficient numerical tests have been performed by comparing simulations with the exact solutions for three-dimensional incompressible flows in rotating periodic and partly periodic cuboids (Poincaré waves in a channel). A simulation of the linear inertial waves in a closed rotating cuboid has been tested against semi-analytical eigenmode solutions. Additionally, a simulation of inertial wave focusing in a hexahedron created by slanting one wall of a cuboid will be discussed.

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