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A 3D explicit Discontinuous-Galerkin groundwater model

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Since human have dug for water, understanding where water goes and where it comes from is a major concern. Unfortunately, due to the complex evolution of the soils, there are no simple answers. Besides major issues to obtain precise underground maps and soil properties, the numerical groundwater flow models encounter many issues. Indeed, the flows cover a broad range of time scales, resulting in highly non-linear governing equation. Many implementations of the Richards equation for variably saturated media have been published to reach a stable, accurate and efficient model, with some good results. Here we propose a variation to these attempts based on an explicit time integration scheme. The non-linear Richards equation for variably water-saturated soils can be expressed as

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K(h) \cdot \nabla(h + z)), \tag{1}$$

$$\theta = f_\theta(h), \tag{2}$$

with h [m] the pressure head, θ [–] the water saturation, z [m] the vertical coordinate, K [ms^{-1}] the conductivity tensor and f_θ [–] the retention curve relationship. This equation is valid both in the saturated zone (SZ) and in the vadoze zone (VZ). Several problems can arise when discretizing this equation with the finite element (FE) method:

- spurious oscillations,
- mass conservation issues,
- convergence issues in very dry cases,
- bad convergence in transient cases due to the high non-linearity of K and f_θ.

Here, we propose novel solutions to tackle these problems in an explicit time discretisation of the Richards equation. As far as we know, all existing models for this equation are using implicit methods. An explicit time integration scheme brings several direct benefits such as simplicity of implementation and coupling, easy and optimal implementation on parallel clusters and the possibility to use slope-limiters or the multi-rate method. The main drawback is the limitation of the time step value by a CFL-like stability condition. This is problematic for small elements and – in our case – near the VZ–SZ interface where the diffusivity became very large. Another problem is that the efficiency of the explicit formulation would be strongly impaired by the need to solve the steady Richards equation in the SZ.

The stability issue is balanced in transient cases by the fact that implicit solvers also have a constraint on the time step to achieve convergence. Indeed, the initial solution has to be close enough to the new one for the iterations to converge. The high non-linearities of the Richards equation can make that constraint quite stringent. The SZ issue is circumvented by using the false transient method approximation. This method consists in adding a time-derivative to the elliptic equations, *i.e.* $\nabla^2 x = s$ becomes $\tau \frac{\partial x}{\partial t} = \nabla^2 x - s$ with τ as small as possible although reducing results in a decrease of the maximum stable time step. This equation can be iterated several times to converge towards a steady-state solution.

We will present the model, and the solutions found for each of the above-mentionned issues. We will compare the explicit-implicit performances with an implicit variable-switching implementation for different test cases.