



## **Second-order Shallow Ice Approximation with Non-linear Rheology: Exploring Validity by Performing Numerical Experiments**

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Until recently, all established large scale numerical ice sheet models were based on the Shallow Ice Approximation (SIA). Employing the SIA, complexity is essentially traded off in favor of computability. However, ice sheets behave critically in zones where SIA based simulations are known to fail or lead to false results. In later years the SIA has therefore been challenged by so called higher order- and Full Stokes-models. Common for those is that they incorporate longitudinal stresses (lacking in the SIA), which are believed to be of crucial importance to the dynamics of ice streams and the grounding line.

While most exact, Full Stokes models are also most costly, hindering therefore an application over essentially unconstrained spatio-temporal domains. Higher order models based on perturbation expansion approach (involving the aspect ratio  $\epsilon$  measuring the shallowness of the ice sheet) are computationally cheap as they do not require iterative solvers: the classical SIA (Hutter 1983, Morland 1984) is the cheapest but also the least accurate in the model hierarchy. Second order SO-SIA implementations are of intermediate complexity, and implementations restricted to Newtonian behavior exist (Mangeny and Califano 1998). However, the generalization to non-Newtonian behavior was hampered by the occurrence of non-linearities and singularities arising from the use of Glen's flow law.

Here, we present an implementation of the SO-SIA for isothermal, steady-state conditions and non-linear rheology. We perform numerical experiments comparing SIA, SOSIA and Full Stokes solutions (from Elmer (<http://www.csc.fi/english/pages/elmer>)) for a model problem where the aspect ratio is varied. We investigate

- whether the order of magnitude of the primary field variables (velocities, stresses) resulting from SIA, SOSIA and Full Stokes computations indeed varies with  $\epsilon$  as suggested by the perturbation expansion approach.
- How large aspect ratios the SIA and SOSIA can be applied for.
- How the singularity arising from the non-Newtonian behavior can be treated appropriately.

### References:

Hutter K (1993), *Theoretical Glaciology; material science of ice and the mechanics of glaciers and ice sheets*, Reidel, Dordrecht, Netherlands, 510 pp.

Mangeny A and Califano F (1998), The shallow-ice approximation for anisotropic ice - formulation and limits, *J Geophys Res* 103(B1), 691-705.

Morland LW (1984), Thermo-mechanical balances of ice sheet flows, *J Geophys Astrophys Fluid Dyn* 29, 237-266.