Intercomparison of different subgrid-scale models for the Large-Eddy Simulation of the diurnal evolution of the atmospheric boundary layer during the Wangara experiment

Dall'Ozzo C., Carissimo B., Musson-Genon L., Dupont E., Milliez M.
CEREA / EDF R&D, 6 Quai Watier 78401 Chatou Cedex
cedric.dall-ozzo@edf.fr

Contexte And Objective

The study of a whole diurnal cycle of the atmospheric boundary layer evolving through unstable, neutral and stable states is essential to test a model applicable to the dispersion of pollutants. Consequently a Large-eddy simulation (LES) of a diurnal cycle is performed and compared to observations from the Wangara experiment (Day 33-34). All the simulations are done with Code_Saturne [1] an open source finite volume CFD code. The synthetic eddy method (SEM) [2] is implemented to initialize turbulence at the beginning of the simulation.

Three different subgrid-scale (SGS) models are tested: the Smagorinsky model [3,4], the Germano and Lilly dynamic model [5,6] and the dynamic Wong and Lilly model [7]. The first one, the most classical, uses a Smagorinsky constant Cs to parameterize the dynamical turbulent viscosity while the other models rely on a variable C. Cs remains insensitive to the atmospheric conditions in contrary to the parameter C determined by the two dynamic models. More, the Wong and Lilly model admits a thermal eddy diffusivity dertimined by a dynamic eddy Prandtl number.

The results are confronted to previous simulations from Basu et al. (2008) [8], who used a locally averaged scale dependent dynamic (LASDD) SGS model, and to RANS simulations with Code_Saturne. The accuracy in reproducing the atmosphere evolution is discussed, especially regarding the night time low-level jet formation. In addition, the benefit of the utilization of a radiative forcing is discussed.

Synthetic Eddy Method (SEM)

The synthetic eddy method (SEM) consists in generating realistic unsteady inflow conditions for large-eddy simulation. The SEM products turbulent structures by adding a perturbation u_i' to the initial mean velocity field :

$$u_i = \bar{u}_i + u_i'$$

The synthetic eddies are created with random position and random intensity. The relation for the velocity perturbation u_i' is:

$$u_i' = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} a\, \varepsilon_j^k S_\sigma^k$$

N is the number of eddies generated, ε_j^k a random parameter (1,-1), S_σ^k is a shape function for the synthetic eddies. Here is a tent function with a compact support [-σ, σ] where σ represent the turbulent length scale:

$$S_\sigma^k = \sqrt{V}\sigma^{-3} \prod_{m=1}^{3} \sqrt{\frac{3}{2}} \left(1 - \left|\frac{x_m - x_m^k}{\sigma}\right|\right)$$

V represents the volume in which eddies are created and a is related to the turbulent kinetic energy k by the relation:

$$a = \sqrt{\frac{2}{3}k}$$

SEM products instantaneous velocity fluctuations necessary for simulations of well developed flows. Generally these fluctuations are generated by the results of amplification of numerical errors in which periodic boundary conditions are defined. Here, they appear earlier in the calculation and permit a realistic LES in the first time steps of the simulation.

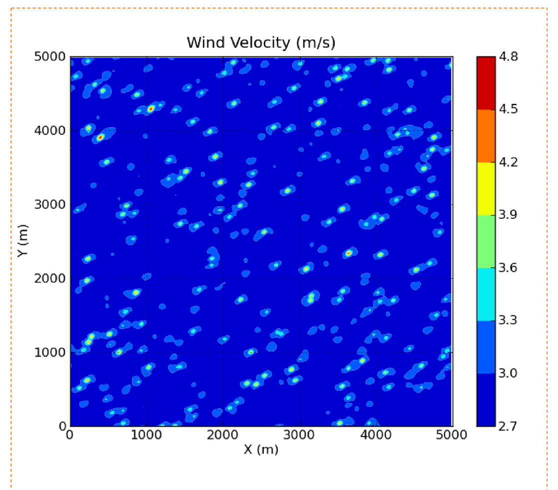

Fig. 1: Horizontal plan of the mean initial velocity field with SEM perturbations.

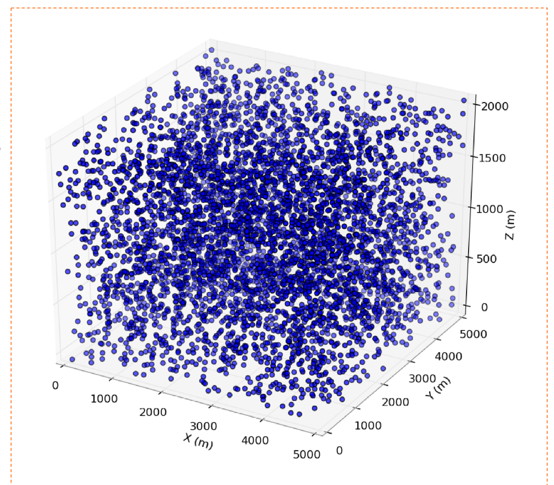

Fig. 2: Random eddy positions in the 3D domain.

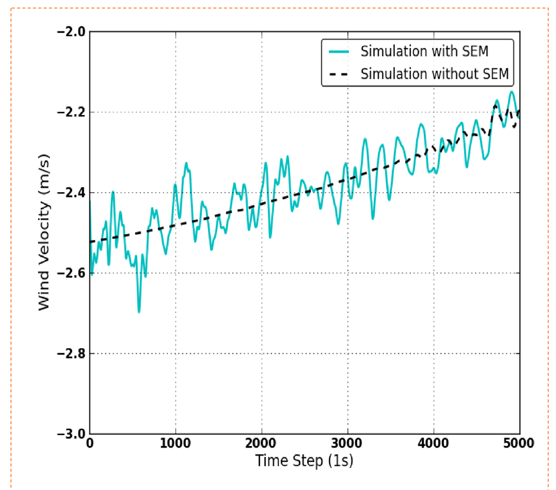

Fig. 3: Temporal evolution of wind velocity probes with and without SEM in the center of the domain.

Subgrid Scale Models Details

The Navier-Stokes equations solved for atmospheric flows in LES are:

$$\frac{\rho}{\langle\rho\rangle}\frac{\partial \bar{u}_i}{\partial t} + \frac{\rho}{\langle\rho\rangle}\frac{\partial(\bar{u}_i\bar{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \nu\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial}{\partial x_i}\tau_{ij} + f_c(\bar{u}_2 - V_g)\delta_{i1} - f_c(\bar{u}_1 - U_g)\delta_{i2} + F_i$$

With ρ the density, u_i the velocity, p the pressure, ν the viscosity, τ_{ij} the SGS stress tensor, Ug and Vg the geostrophic wind, f_c the Coriolis parameter and the F_i forcing terms. <> defines a horizontal plane average.

Smagorinsky Model

The Smagorinsky model is the most popular subgrid-scale (SGS) model. It consist in modelizing the SGS stress tensor τ_{ij} as follows:

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j = 2\nu_t \bar{S}_{ij}$$

With S_{ij} the resolved strain rate tensor: $\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$

The turbulent viscosity used depends on the constant Smagorinsky coefficient C_s, the resolved strain rate tensor S_{ij} and the grid filter scale Δ:

$$\nu_t = (C_s\Delta)^2\sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$$

Germano & Lilly dynamic Model

The Germano model admits a second higher filter scale $\tilde{\Delta} = 2\bar{\Delta}$. Two SGS stress tensors τ_{ij} and T_{ij} are defined:

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j \qquad T_{ij} = \widetilde{\overline{u_i u_j}} - \tilde{\bar{u}}_i\tilde{\bar{u}}_j$$

From these two definitions it's possible to construct the the Germano identity L_{ij} relation and, using the least square method, yield to C:

$$L_{ij} = T_{ij} - \tilde{\tau}_{ij} = \widetilde{\bar{u}_i\bar{u}_j} - \tilde{\bar{u}}_i\tilde{\bar{u}}_j$$

If M_{ij} is defined as follows:

$$M_{ij} = 2\bar{\Delta}^2\left(\widetilde{|\bar{S}|\bar{S}_{ij}}\right) - \left(2\tilde{\Delta}^2\left(|\tilde{\bar{S}}|\tilde{\bar{S}}_{ij}\right)\right)$$

After a local average the coefficient C is:

$$C = \frac{\langle L_{ij}M_{ij}\rangle_{local}}{\langle M_{mn}M_{mn}\rangle_{local}}$$

Wong & Lilly Model

The Wong and Lilly dynamic model is close to the previous one. The most important difference is that the thermal eddy diffusivity, here, is calculated with a dynamical Prandtl number determination:

$$C = \frac{\langle L_{ij}\tilde{\bar{S}}_{ij}\rangle}{2\bar{\Delta}^{4/3}(1-(\tilde{\Delta}/\bar{\Delta})^{4/3})\langle\tilde{\bar{S}}_{mn}\rangle} \qquad Pr = \frac{\langle R_{\theta i}(\partial\tilde{\bar{\theta}}/\partial x_i)\rangle}{2\bar{\Delta}^{4/3}(1-(\tilde{\Delta}/\bar{\Delta})^{4/3})\langle(\partial\tilde{\bar{\theta}}/\partial x_j)^2\rangle}$$

$R_{\theta i}$ is defined similarly to L_{ij}:

$$R_{\theta i} = T_{\theta i} - \tilde{\tau}_{\theta i} = \widetilde{\bar{\theta}\bar{u}_i} - \tilde{\bar{\theta}}\tilde{\bar{u}}_i$$

Subgrid Scale Models Comparison

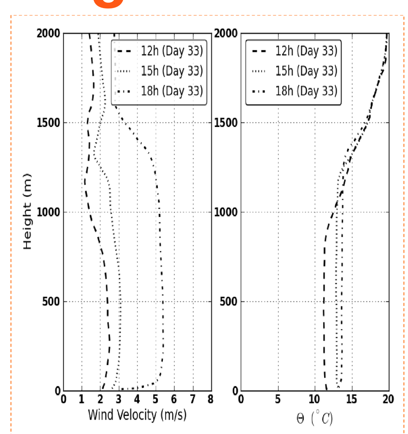

Fig. 4: Basu et al. in daytime

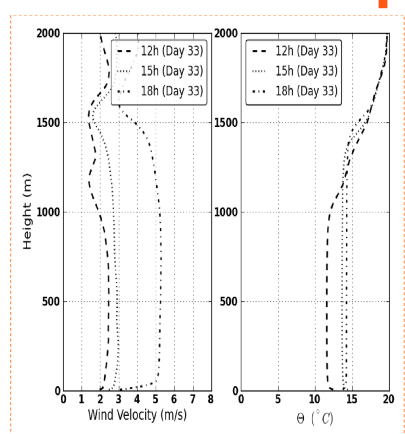

Fig. 5: Smagorinsky in daytime

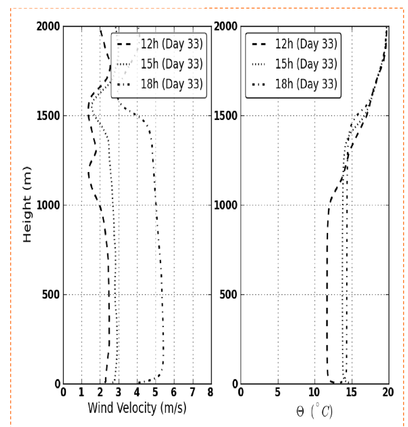

Fig. 6: Germano & Lilly in daytime

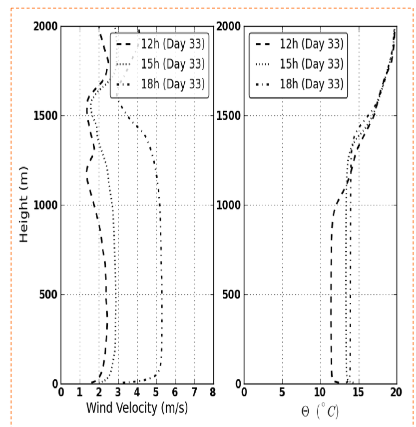

Fig. 7: Wong & Lilly in daytime

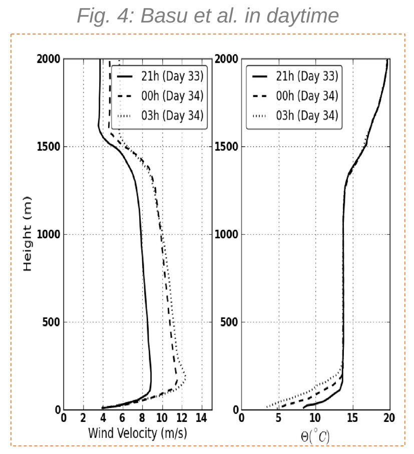

Fig. 8: Basu et al. in nighttime

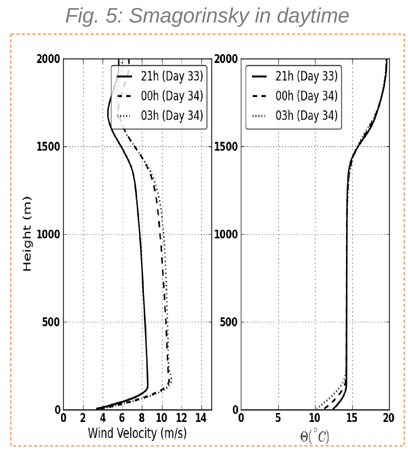

Fig. 9: Smagorinsky in nighttime

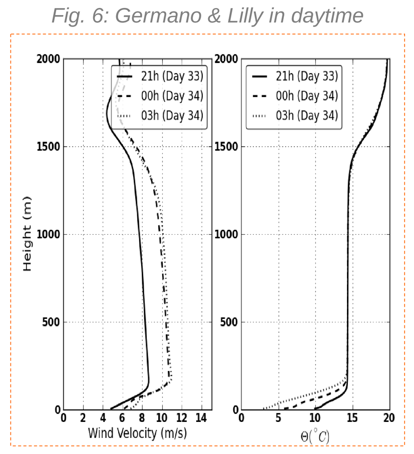

Fig. 10: Germano & Lilly in nighttime

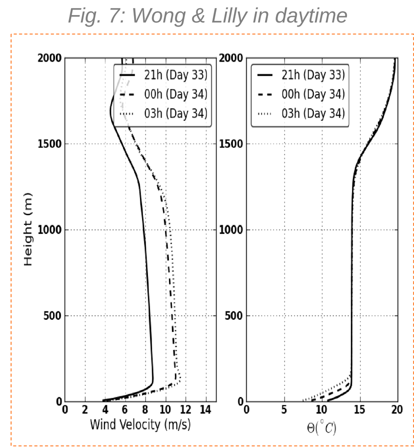

Fig. 11: Wong & Lilly in nighttime

Radiative Heat Transfer Forcing

Radiative heat forcing is tested for the different SGS models. It takes into account the infrared and solar transfers during all the diurnal cycle. The heating and cooling effect are considered by adding forcing terms into the potential temperature equation.

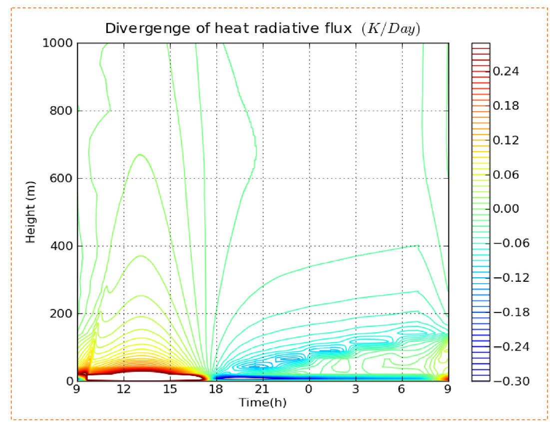

Fig.12 : The divergence of the radiative flux taken into account as a forcing term during the diurnal cycle (solar-infrared).

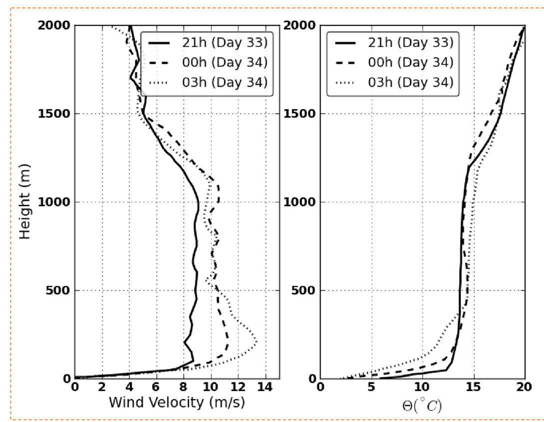

Fig.13 : Observed nighttime wind velocity and potential temperature during the Wangara experiment.

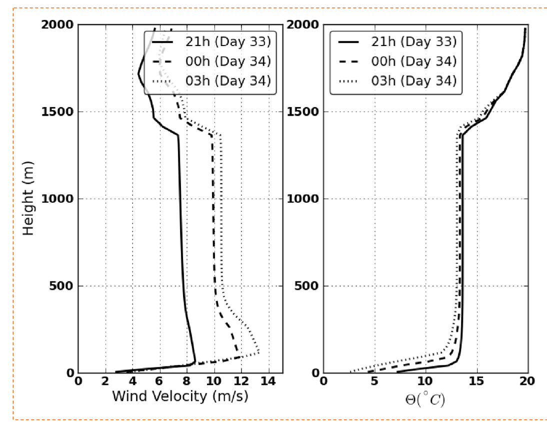

Fig.14 : Nighttime wind velocity and potential temperature for a RANS model with a heat radiative forcing.

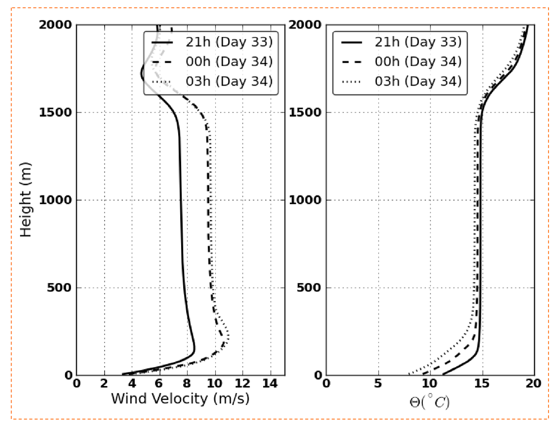

Fig.15 : Nighttime wind velocity and potential temperature for a Smagorinsky SGS model with a heat radiative forcing.

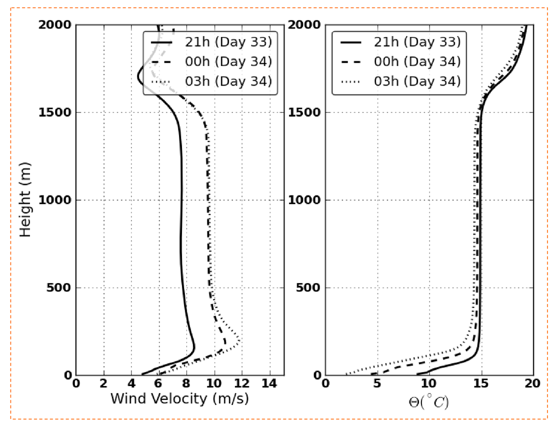

Fig.16 : Nighttime wind velocity and potential temperature for a Germano and Lilly dynamic SGS model with a heat radiative forcing.

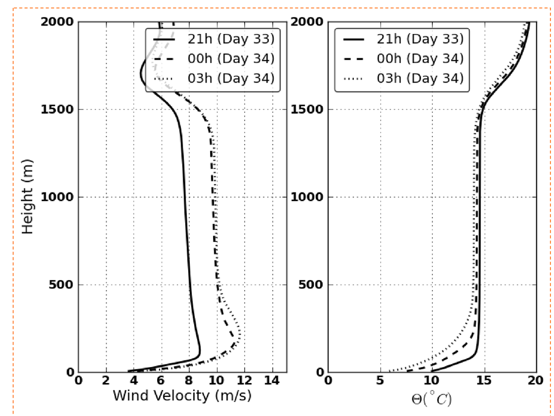

Fig.17 : Nighttime wind velocity and potential temperature for a Wong and Lilly dynamic SGS model with a heat radiative forcing.

The radiative heat transfer forcing appears to be essential to simulate a right low level jet (LLJ). Without this term the model is unable to develop an important LLJ. To reach the Basu and al. results obtained with a lagrangian dynamic SGS model, the only solution appears to take into account heat radiative forcing . LES can reproduce LLJ shape close to Basu and al. results but RANS simulation is more efficient to reconstruct mean profiles similar to observed measures.

Conclusion

✓ **The synthetic eddy method products realistic unsteady inflow conditions which help velocity fluctuations to appear earlier in the simulation.**

✓ **In daytime differences between the three SGS models are quite small. The large eddies present the afternoon are well modeled by all models. Here, the importance of the SGS models is relative.**

✓ **During the nighttime, due to the weaker turbulent structures, the gap between the different SGS models becomes more important in the 500 meters above the surface. The importance of SGS dissipation is more important during stable periods and results are more linked to the type of SGS models used.**

✓ **The radiative heat forcing, used with several SGS models, is essential for the simulation of well developed low level jet in the first meters of the atmospheric boundary layer .**

References

[1] Archambeau F., Mehitoua N., Sakiz M., "Code_Saturne: a finite volume code for the computation of turbulent incompressible flows", International Journal on Finite Volumes 1(1), 2004.

[2] Jarrin N., Benhamadouche S., Laurence D., Prosser R., "A synthetic-eddy-method for generating inflow conditions for large-eddy simulations", International Journal of Heat and Fluid Flow, 27, 585-593, 2006.

[3] Smagorinsky J., "General circulation experiments with the primitive equations", Monthly Weather Review, 91, 99-164, 1963.

[4] Lilly D. K., "On the application of the eddy viscosity concept in the inertial sub-range of turbulence", National Center For Atmospheric Research, Manuscript 123, 1966.

[5] Germano M., Piomelli U., Moin P. and Cabot W. H., "A Dynamic Subgrid-Scale Eddy Viscosity Model", Physics of Fluids A, 3(7), 1760-1765, 1991.

[6] Lilly D. K., "A proposed modification of the Germano subgrid-scale closure method", Physics of Fluids A, 4(3), 633-635, 1991.

[7] Wong V. C. and Lilly D. K., "A comparison of two dynamic subgrid closure methods for turbulent thermal convection", Physic of Fluids, 6(2), 1016-1023, 1994.

[8] Basu S., Vinuesa J. F. and Swift A., "Dynamical LES modeling of a diurnal cycle", Journal of Applied Meteorology and Climatology, 47, 1156-1174, 2008.