# Similarity scales in the stable boundary layer : a test against data

**F. Tampieri** (1), C. Yagüe (2) and S. Viana(3)

(1) CNR ISAC, Bologna, Italy (f.tampieri@isac.cnr.it ). (2) Departamento Geofísica y Meteorología. Universidad Complutense de Madrid, Spain. (3) Agencia Estatal de Meteorología, Delegación Territorial de Cataluña, Barcelona, Spain

## 1. INTRODUCTION

- Turbulence is the stable boundary layer is affected by local conditions (surface fluxes and the thermal and dynamic structure of the atmosphere) as well as by non local effects like gravity waves or horizontal inhomogeneities.
- In the perspective of a local description, different similarity scales have been proposed in the literature, and summarized for instance by [1], [2].
- Using first and second order moments of velocity and temperature data obtained at CIBA (see Fig. 1) during the SABLES98 campaign [3], different formulations have been compared to explore the range of applicability and possible shortcomings.

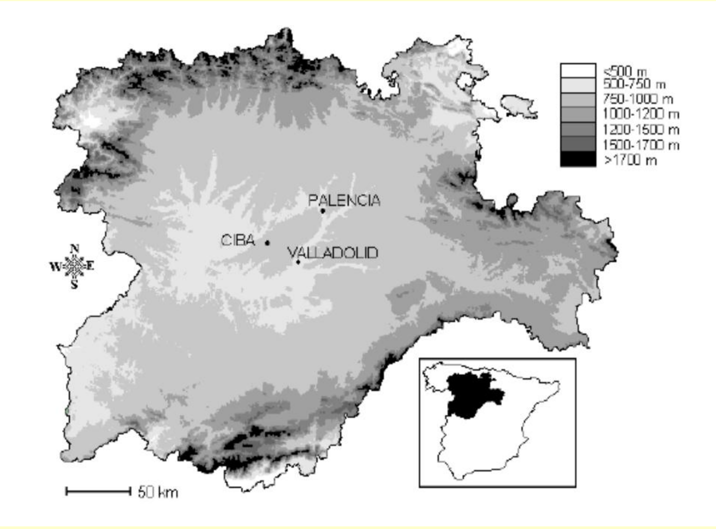


**Figure 1:** Location of the CIBA, SPAIN

## 2. AVAILABLE DATA AND ANALYSIS

- The data refer to 5 min intervals.
- Variances of temperature and velocity and turbulent fluxes from sonic anemometers at three heights (5.8, 13.5, 32 m) .
- Mean horizontal velocity from cup anemometers at five heights (3, 10, 20, 50 and 100m).
- Mean temperature from thermocouples at 15 levels, from 0.22 to 50 m.

- To avoid low frequency bias on the second order estimates, the MultiResolution Flux Decomposition (MRFD) has been used. MRFD is an orthogonal decomposition used for computing variances and turbulent fluxes [4], [5]. For instance, the series of vertical velocity and temperature of $2^n$ points are recursively filtered by moving average of decreasing lenght (here n = 14). Covariances are computed for each cycle.

- The differences between consecutive covariances represent the contribution to the total flux from structures of different time scales. Every MR heat cospectra is fitted to a 5-th order polynomial. The timescale of the spectral gap is estimated according to a criterion based on the first occurrence of a zero crossing or inflection point after the maximum of downward turbulent heat transfer.

## 3. SIMILARITY SCALES AND THE RICHARDSON NUMBER

- From Monin-Obukhov Similarity Theory (MOST) scales based on fluxes can be defined:

$$u_*(z) = \left(\overline{u'w'}^{\,2} + \overline{v'w'}^{\,2}\right)^{\frac{1}{4}} \quad (1)$$

$$\vartheta_*(z) = \frac{\overline{w'\vartheta'}(z)}{u_*(z)} \quad (2)$$

$$\Lambda(z) = -\frac{\vartheta_{00}\, u_*^{3}(z)}{kg\,\overline{w'\vartheta'}(z)} \quad (3)$$

- According to [2] scales based on gradients of mean quantities can be defined:

$$u_s(z) = lN = l\left(\frac{g}{\vartheta_{00}}\frac{d\bar{\vartheta}}{dz}\right)^{\frac{1}{2}} \quad (4)$$

$$t_s = l\,\frac{d\bar{\vartheta}}{dz} \quad (5)$$

$$l = \frac{kz}{1+\dfrac{kz}{l_0}} \quad (6)$$

- To avoid estimates of gradients of mean quantities based on polynomial interpolation of measurements we shall use the bulk Richardson number based on differences of mean values at different heights.:

$$R_b(z_1,z_2,z_3,z_4) = \frac{g}{\vartheta_{00}}\,\frac{\Delta\bar{\vartheta}}{z_2-z_1}\,\frac{(z_4-z_3)^2}{(\Delta\bar{u})^2+(\Delta\bar{v})^2} \quad (7)$$

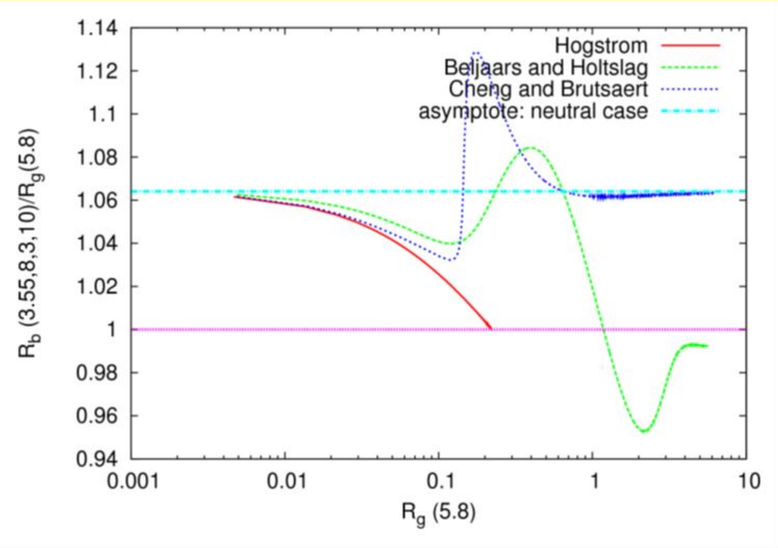


**Figure 2.** Ratio of the bulk Richardson number and the gradient Richardson number, computed from three parameterizations (Hogstrom, Beljaars and Holtslag, Cheng and Brutsaert) and for heights typical for this data set.

- It can be shown that using common parameterizations for profiles, *Rb* differs less than 10% from the gradient Richardson number, for heights typical of this experiment.

## 4. RESULTS

- Figure 3 illustrates the ambiguity in the use of both Obukhov length and Richardson number: data points with a given *Rb* may have *L* different by one order of magnitude, especially for the large stability. This is not captured by parameterizations. Note from the figure that SBL with TKE increasing with height are typically more stable.
- Figure 4 shows Velocity difference normalized according to the equation::

$$\frac{\Delta u}{u_*(z_m)} = \left[\frac{1}{k}\log\left(\frac{z_4}{z_3}\right) + \frac{z_4 - z_3}{l_0}\right]\left(1+300\,R_g^{\,2}\right)^{\frac{3}{4}} \quad (8)$$

Although the scatter of the data is large, the overall behaviour agrees well with Sorbjan formulation up to *Rb*<0.5. For larger values the velocity difference decreases. The representation in terms of flux scaling puts into evidence that high *Rb* do not necessarily correspond to high z/L; moreover, the data mainly lie between the log-linear relation (Hogstrom) and the Beljaars and Holtslag expression.

- The equivalent for temperature variance is presented in the figures 6 and 7.

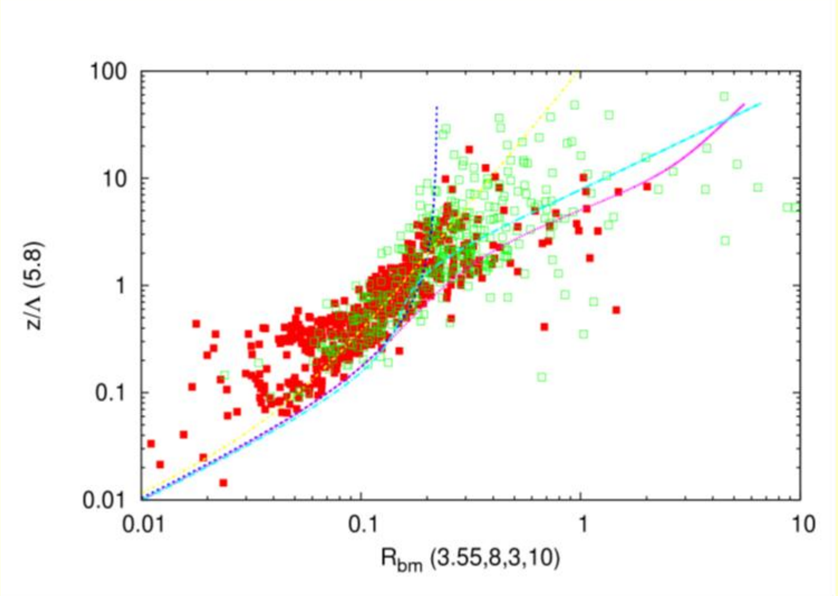


**Figure 3.** z/Λ vs. *Rb*(3.55, 8, 3,10) from the observations. Here Sorbjan et al. (2010), Eq. 24 is used. Full squares: SBL with TKE decreasing with height; open squares: TKE increasing with height.

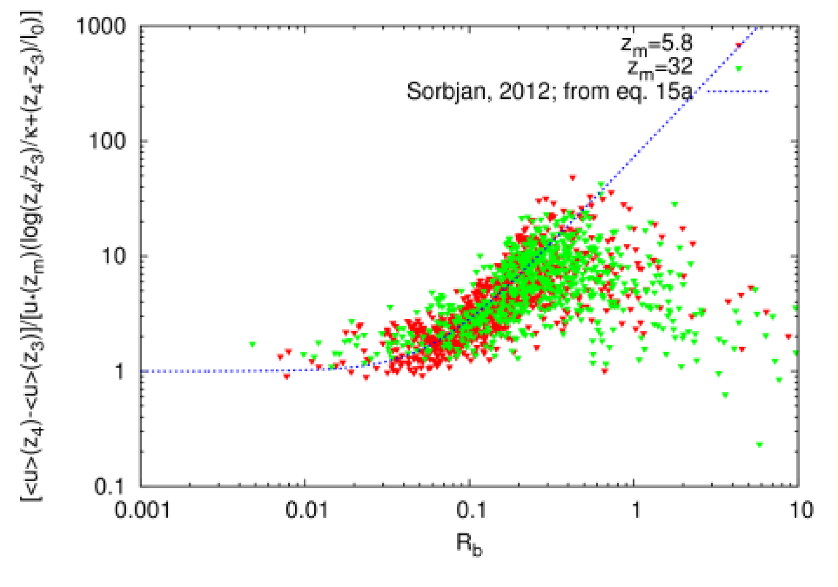


**Figure 4.** Velocity difference normalised according to Eq.8 as function of bulk Richardson number.

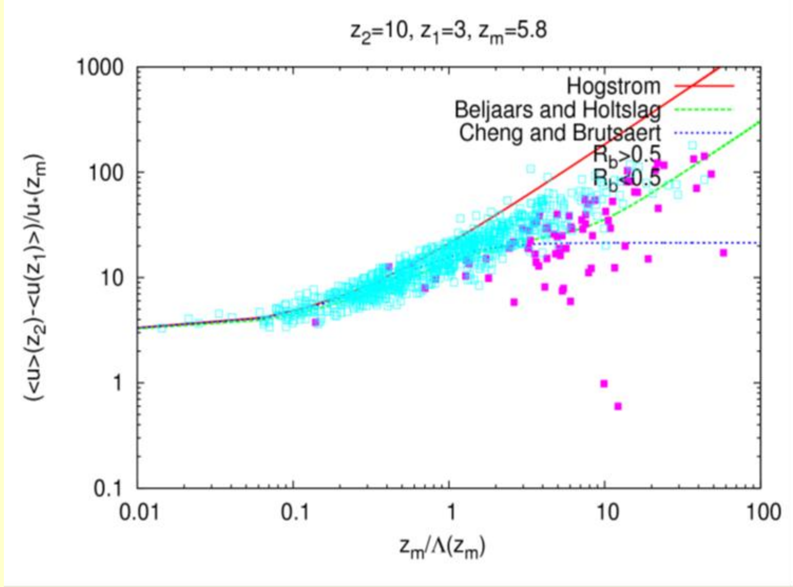


**Figure 5.** : Velocity difference normalised with u* as function of z/Λ. Open squares: data with *Rb*<0.5 (moderately stable), full squares: data with *Rb*>0.5.

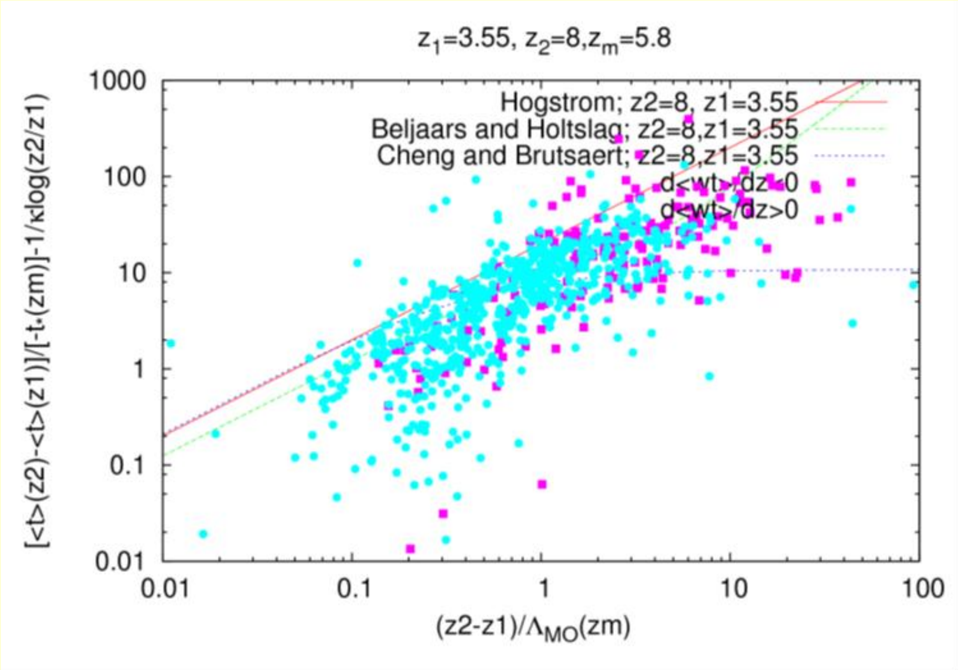


**Figure 6.** : Temperature difference normalised over t* as function of z/Λ.

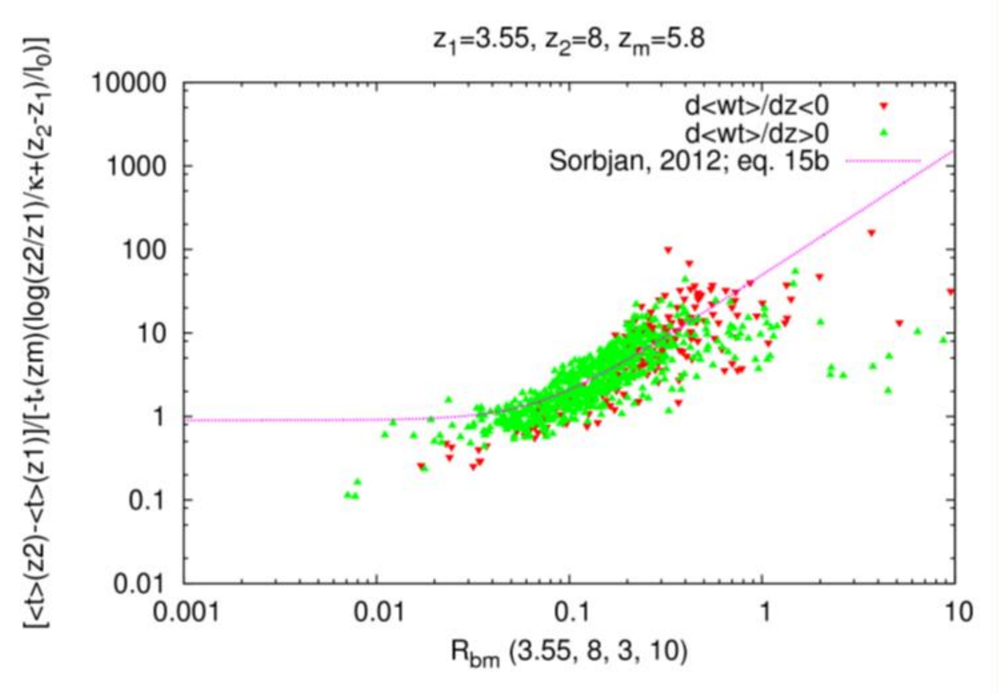


**Figure 7.** Temperature difference normalised according to Sorbjan formulation, as function of the bulk Richardson number.

## 5. SUMMARY AND CONCLUSIONS

- Scaling based on gradients of mean quantities are suitable to describe the local similarity of turbulence variables in SBL with moderate stability (*Rb*<0.5); the same holds for scaling based on fluxes (according to MOST) for z/L<1. Departures for large stability hinder the possibility to define accurate similarity functions.
- In spite of the different mechanisms producing turbulence, local similarity is verified both for cases with fluxes decreasing with height and for cases with increasing fluxes.
- It appears that each scaling is not able to fully describe even the simplest case (decreasing fluxes), suggesting the need to use an extended similarity which includes more parameters (besides *L* or *Rb*).

## 6. REFERENCES

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## 7. ACKNOWLEDGMENTS

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