EGU2012-4944

Functional Friction Networks

H.O. Ghaffari and R. P. Young

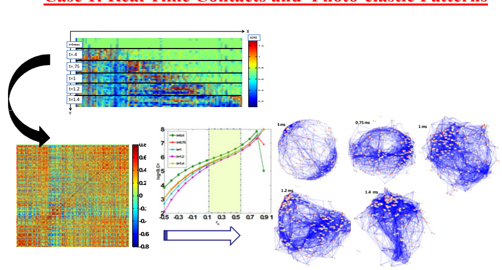
Department of Civil Engineering and Lassonde Institute, University of Toronto, Toronto, ON, Canada

E-mail: h.o.ghaffari@gmail.com

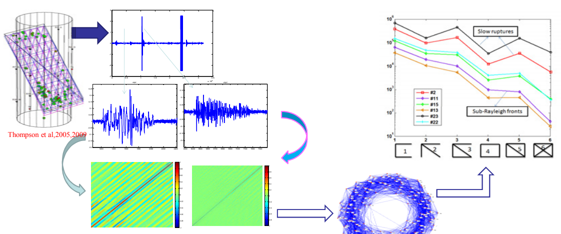
➢ **Abstract** : We have developed different network approaches to complex patterns of frictional interfaces . Network theory is a fundamental tool for the modern understanding of complex systems in which, by a simple graph representation, the elementary units of a system become nodes and their mutual interactions become links. With this transformation of a system to network space, many properties about the structure and dynamics of the system itself can be inferred. We map the real-time net contact areas, photo-elastic patterns and acoustic waveforms to network configurations while we use similarity measures to link the nodes. In other words, we follow the possible collective deformation of contact areas as well as the characteristics of correlated elements. Here, we analyze the dynamics of static friction. We found, under the correlation measure, the fraction of triangles correlates with the detachment fronts. Also, for all types of the loops (such as triangles), there is a universal power law between nodes' degree and motifs where motifs frequency follow a power law. This shows high energy localization is characterized by fast variation of the loops fraction. Moreover, we confirmed that slow ruptures generally hold small localization, while regular ruptures carry a high level of energy localization. We proposed that assortativity, as an index to correlation of node's degree, can uncover acoustic features of the interfaces.

➢ **Procedure and Conclusions :** We map the acoustic waveforms –obtained from saw-cut fault embedded in Westerly granite- and real-time contacts as well as photo-elastic into network spaces[1,3,4]. The networks are constructed based on 1) time-delay method (standard-delay-coordinate embedding method) and 2) thresholded-closeness metric as well as the correlation of pairs . We calculate several characteristics of the obtained network and find out motif distributions follow three distinct patterns ,indicating at least long-frequency events and sub-Rayleigh ruptures. We approximate the rupture speed with using evolution of networks degree . Remarkably, the patterns of motifs from this case is comparable with real-time contact networks (with the same or mentioned approaches in networks building). Slow fronts show higher distribution rank of loops rather than regular shocks. It seems for both cases sub-Rayleigh events have less homogeneity while slow fronts follow slightly more homogeneous motifs ranks [5]. We also found some parameters of the acoustic friction networks can be used to classify the type of the rupture's speed . A universal trend is observed in ground motion of fronts ,confirming the recent observation s on PMMA [1,2]. Generally ,we proposed that the complexity of frictional interfaces are the results of collective behaviour of the system's elements and equally can be captured with functional friction networks .

Case 1: Real Time Contacts and Photo-elastic Patterns



Case 2: Waveforms Analysis -Smooth Fault



standard delay-coordinate embedding method

➢ Left panel: Methodology to extract friction networks from real t-time contacts: Evolution of real-time contacts through 6 time-windows in X-Y space of an interface [1]. Each pixel corresponds with net contact area (contact areas data set courtesy of J.Fineberg); visualization of the correlation matrix and achieved friction networks through 5 time-windows. natural logarithmic variation of mean betweenness centrality (B.C) with truncation value. The indicated interval with arrows shows the best possible threshold level where the minimum variation of log<B.C>occurs (the most stable-dominant structures). <.. .> indicates average over all nodes (i.e. aperture). Right Panel : The methodology used to extract friction networks from the acoustic signals of the evolution of a smooth fault (interface). The figure shows two different acoustic waves corresponding to regular shear and slow rupture. To analyze waveforms and to construct networks, the standard-delay time method was used in conjunction with the thresholded adjacency matrix [5]. The distribution of the motifs may be used as arough estimation of rupture type ;they also follow a universal trend which is compatible with aperture and contact friction networks; Right panel : correlation patterns of transferred time-series to extra-dimension; The motifs ranks over waveforms show three distinctive patterns ,corresponds to slow rupture, sub-Rayleigh and noise-like waveforms. Remarkably, there is a same trend in waveform-based motifs of networks and real-time contact measures . Also, we observed in slow-rupture motifs are distributed uniformly rather than regular fronts-motifs

Damage Evolution and Friction Law with Complex Networks

$$T(k) \sim k^{\beta},$$

$$\frac{\partial \theta_i(t)}{\partial t} = a\frac{\partial k_i}{\partial t} + b\frac{\partial T_i}{\partial t}$$

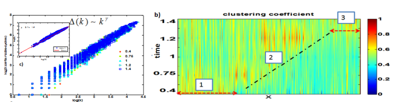
$$\frac{\partial \theta_i(t)}{\partial t} = \frac{\partial k_i}{\partial t}(a + b\beta k_i^{\beta-1})$$

$$\frac{\partial T_i(k)}{\partial t} \sim m\frac{T_i}{\sum_{T_j}},$$

A-B model for the growth of network

$$\frac{\partial k_i}{\partial t} \sim \frac{m}{\beta}\frac{k_i}{\sum_{k_j^{\beta}}}$$

$$\frac{\partial \theta_i(t)}{\partial t} = \frac{2bmk_i^2}{\sum_{k_j^2}} + \frac{amk_i}{\sum_{k_j^2}}$$



$$\frac{\partial f_s}{\partial t} \sim -\left(\frac{\partial C(k)}{\partial t}\right) \longrightarrow f_s \sim \varepsilon \ln k(t)$$

$$\frac{\partial^2 C(x,t)}{\partial x \partial t} \sim -1/v_{front} \longrightarrow 1/v_{front} \sim -\chi(k)\frac{\partial^2 \theta(x,t)}{\partial x \partial t}$$

$$v_{front}\frac{\partial^2 \theta(x,t)}{\partial x \partial t} + \frac{\partial v_{front}}{\partial t} = \frac{-1}{D_c}$$

State variable evolution in terms of local and global structures of corresponding networks.

~ Non-linear kernel for the growth /decay of network :

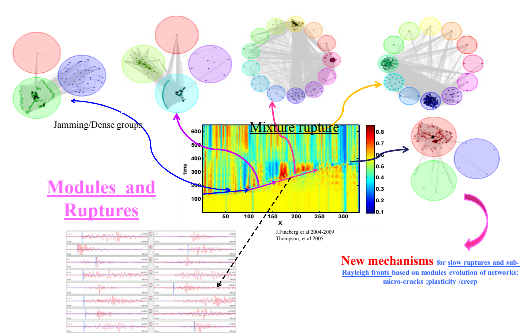
Gel-like evolution of the state parameter/Condensation of the sate variable

Why friction complexity somehow can be captured with the state variable?

Ghaffari& Young NGP ,2012

Ghaffari& Young ARMA 2012

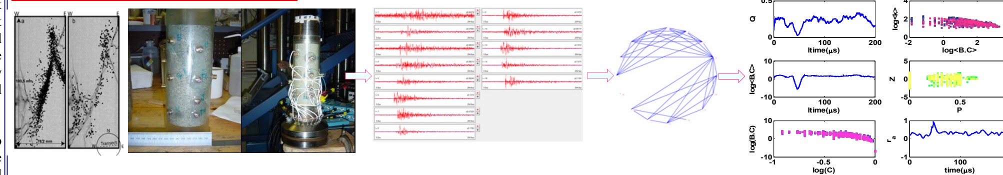
Self-Organized Criticality and Rupture Transitions



New mechanisms for slow ruptures and sub-Rayleigh fronts based on modules evolution of networks: micro-cracks ;plasticity /creep

➢ Left panel: Formulation of friction-network model based on Albert-Barabasi and Non-linear kernel models based on the 1)scaling of triangles in obtained networks with number of similar profiles (node's degree) ,expressed with power law relation with universal exponent (~2) and 2) clustering coefficient as a fraction of triangles reveals the relatively precise rupture speed :three distinctive rupture speeds is compatible with mean contact area ;1 , 3 corresponds with sub-Rayleigh rupture and 2 is slow rupture ;
Right Panel :configurations of groups in sub-Rayleigh and slow ruptures and their transitions ; the un-stable slow rupture scales with increasing communities and then possible less energy localization ;however fast rupture correlates with "jamming" of the nodes in tight communities . The obtained results show synchronization level in slow ruptures occurs is higher than other types of ruptures in terms of maximum eigenvalue of laplacian matrix. The presented observations provide further ideas to model rupture transitions in terms of network space.

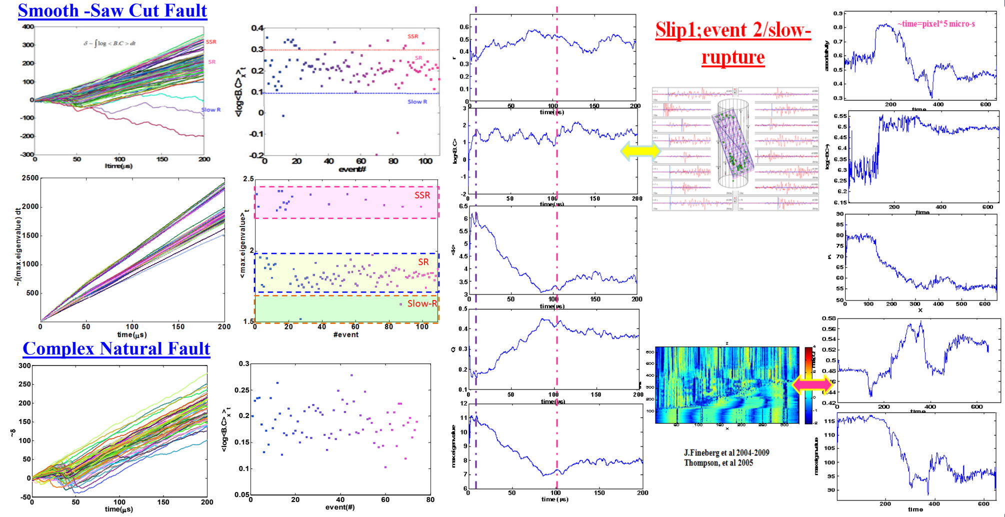
Case 3: Acoustic-Friction Networks



➢ With simple closeness metric ,one can extract acoustic-friction network from acoustic-waveforms. Here, we explore the network space of the recorded waveforms through 13-16 transducers when a saw cut-fault or a complex natural fault was evolving . The characteristics of the acoustic-friction networks (for example right panel) are comparable with photo or real-time contact friction networks. The betweenness centrality (B.C)-clustering coefficient parameter space show a universal collapse of the waveforms as well as the distribution of local energy flow (~B.C). Our observations over photo-elastic ,real-time contacts ,aperture patterns of the frictional interfaces and acoustic friction network patterns showed that BC-C parameter space has a general gamma function distribution. The waveforms cover a certain regions of the with-in module degree and participation coefficient (P-Z space) which are completely different from random networks (i.e., null model). We compared the mean of local flow energy with the recorded acceleration [4] and found that the ground motion features can be captured with the <B.C> . The recent approach tested on different types of the ruptures over photo-elastic friction networks. The possible estimation of quasi-slip of a rupture front can be followed by the integration of <B.C>. Q and r_a stands for maximum modularity and assortativity, respectively.

Smooth -Saw Cut Fault

Complex Natural Fault



➢(Left panel): The possible estimation of quasi-slip of a rupture front can be followed by the integration of <B.C>. With this integration of local flow energy in acoustic-friction networks ,we could recognize three different types of the ruptures in smooth-fault experiment (slow rupture, sub-Rayleigh fronts and super-shear fronts); The big percent of the recorded waveforms showed regular ruptures. Average value of maximum eigenvalue of the constructed friction networks represent nearly the same classification of <B.C> and motifs ranks. A universal pattern of quasi-slip index over 109 events in smooth-fault and complex natural fault is compatible with recent observation of real-time contact measures and precise measure of the slip[1]. Right panel :example of slow rupture with dynamic contact and acoustic friction networks: Comparing acoustic-friction networks with contact-friction networks: Transferring 1D contact areas [1] to networks ;(a) clustering coefficient versus time (~4ms),(b) real-time contact areas nearly show three distinct regimes : slip stage ;aging/sliding (c) each element of interface-network is mapped into modularity space (participation factor of each edge and within module degree)-For different case studies we found elements of interfaces occupy certain regions of P-Z space ;(d) node's degree versus time (~4ms) nearly follow contact areas variation-This yields we may write state parameter in terms of networks variables ;(e) assortativity coefficient versus time (~4ms) shows all of networks are assortative ; transition to new rupture speed dramatically change assortativity- In Transition to aging step ,as if real time contacts are changing less than 3-5% how assortativity changes 15-25%;(f) maximized Q as an index to modularity versus time ;(g) maximum eigenvalues of Laplacian of node's degree against time indicate possible synchronization of elements of frictional interfaces;(h) Betweenness Centrality –as a measure of a node's centrality in a network-indicate clear rupture transitions and possible periodic nature of interface (characteristic length in friction law).

➢**References :**
1-S. Rubinstein, G. Cohen, and J. Fineberg, Nature 430, 1005 (2004); S.M.Rubinstein, G.Cohen, and J.Fineberg, J. Phys. D: Appl. Phys. 42, 214016, (2009).
2-O Ben-David, SM Rubinstein, J Fineberg, Slip-stick and the evolution of frictional strength ,Nature 463 (7277), 76-79.
3-Thompson, B.D., D.A. Lockner and R.P. Young, Premonitory acoustic emissions and stick slip in natural and smooth faulted Westerly granite,J. Geophysical Research, Vol 114, B02205, doi: 10.1029/2008jb005753, 2009; Thompson, B.D,R.P. Young, D.A. Lockner, Observations of Premonitory Acoustic Emission and Slip Nucleation during a Stick Slip Experiment in Smooth Faulted Westerly Granite. Geophysical Research Letters, 32, Art. No: L10304, 2005.
4-Schubnel, A., S. Nielsen, J. Taddeucci, S. Vinciguerra, and S. Rao (2011), Photo-acoustic study of subshear and supershear ruptures in the laboratory, Earth Planet. Sci. Lett., v. 308(3-4), pp. 424-432, doi: 10.1016/j.epsl.2011.06.013..
5-Ghaffari H.O., and Young R.P, Nonlin. Processes Geophys., 19,(2012),215-225; H.O.Ghaffari, A.Nabovati, M. Sharifzadeh, and R.P.Young, http://arxiv.org/abs/1103.1124 ; H.O.Ghaffari and R.P.Young, Motifs of Networks from Shear Fractures http://arxiv.org/abs/1108.5078 ; H.O.Ghaffari and R.P.Young, Network –Configurations of Dynamic Friction Patterns (ARMA 2012)

➢**Acknowledgment :**
We would like to acknowledge and thank **J. Fineberg** (The Racah Institute of Physics, Hebrew University of Jerusalem, Israel) ,**S. Maegawa** (Graduate School of Environment and Information Sciences, Yokohama National University, Japan) ,**B.D.Thompson** (University of Toronto) and **S.Nielsen** (Istituto Nazionale di Geofisica e Vulcanologia) who provided the data set employed in this study.