Plasmoid Dynamics in Resistive MHD Simulations of Magnetic Reconnection

Ravi Samtaney

Mechanical Engineering, PSE and MCSE Divisions

King Abdullah University of Science and Technology

N. F. Loureiro

IPFN Portugal

A. A. Schekochihin

University of Oxford

D. A. Uzdensky

University of Colorado, Boulder

EGU 2012

April 25, 2012

KAUST

Resistive MHD reconnection: an unsolved problem

- Resistive MHD is the simplest framework with which reconnection can be described.
- Until ~5 years ago, it was believed that resistive MHD reconnection was accurately described by the *Sweet-Parker (SP)* model.
- ***In most systems of interest, S>>1: SP model is too slow to explain observations. However, collisionless reconnection is fast***

- However:
  1. ***Not all plasmas are collisionless***. *Many astrophysical environments are very high density – solar chromosphere, interstellar medium, inside stars and accretion disks, etc – so reconnection layer is collisional and resistive MHD should apply*. ***Can reconnection be fast in these environments?***
  2. *Numerical evidence* ***for current sheet instability to secondary islands*** *(plasmoids) had been around for quite a while (e.g., Park '84, Steinolfson '84, Biskamp '86, Loureiro '05)*
  3. *Background turbulence may drastically affect the reconnection process, make it fast (Matthaeus & Lamkin '86, Lazarian & Vishniac '99, Kowal et al., '09, Loureiro et al., '09)*

KAUST

Current sheet instability: threshold

Linear resistive MHD theory (Loureiro et al. PRL 2005) predicts:

$$\gamma_{\max}\tau_A \sim S^{1/4}$$ **Super Alfvenic growth**

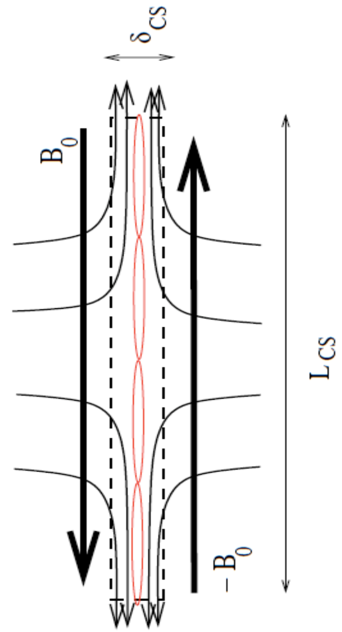
$$k_{\max}L_{CS} \sim S^{3/8}$$ **Plasmoids galore**

- To a good approximation, outflows in the CS are linear (Yamada *et al. '00*, Uzdensky & Kulsrud *'00*):

$$v_y \approx V_A y / L_{CS}$$

- For any perturbation to grow, its growth rate needs to exceed the shearing rate:

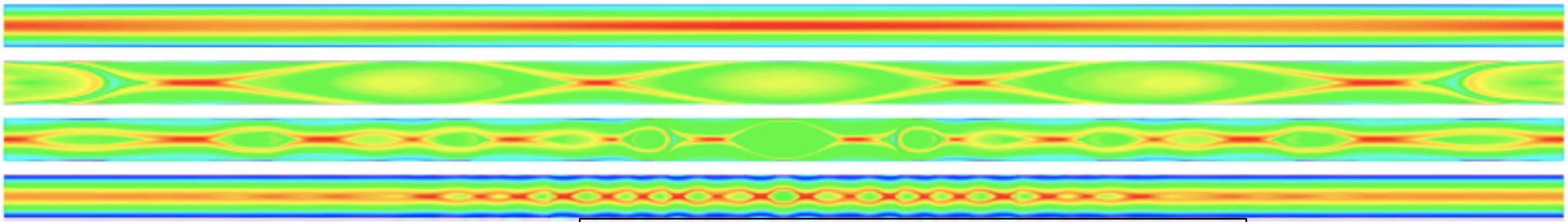
$$\gamma\tau_A >> 1 \Rightarrow S^{1/4} >> 1$$



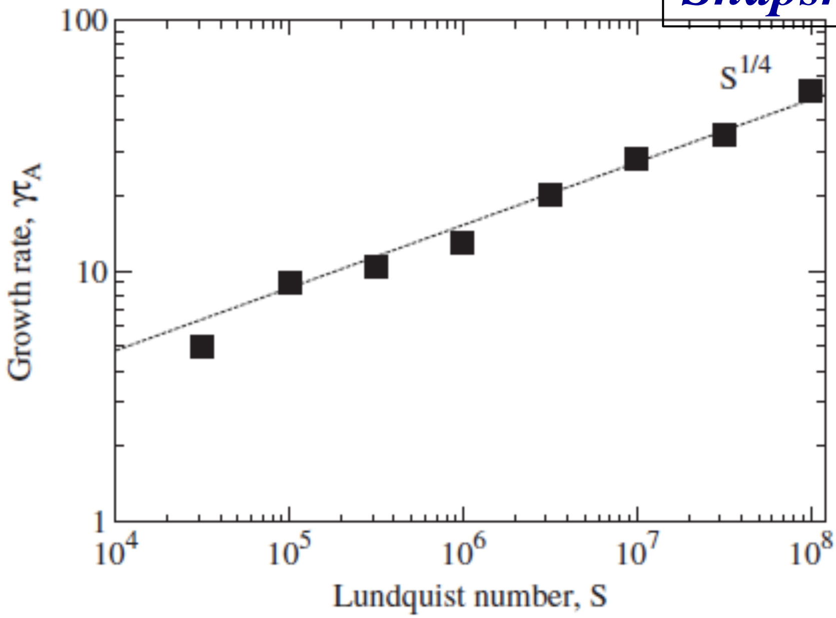
→ ***Critical threshold for instability:*** $S_c \sim 10^4$

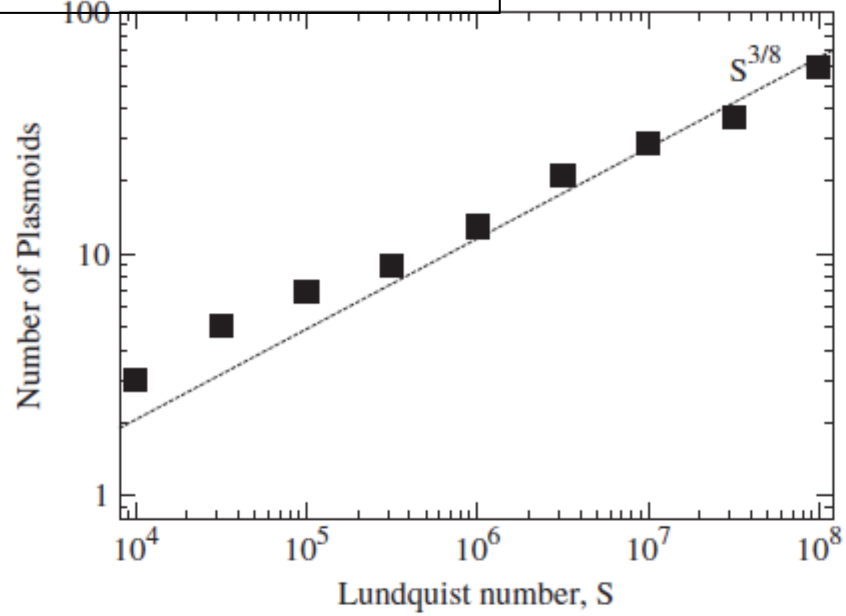
Numerical confirmation of linear theory

Numerical simulations confirm scalings predicted by linear theory (Samtaney *et al*., PRL '09).



Snapshots for S=10⁴, 10⁵, 10⁶, 10⁷



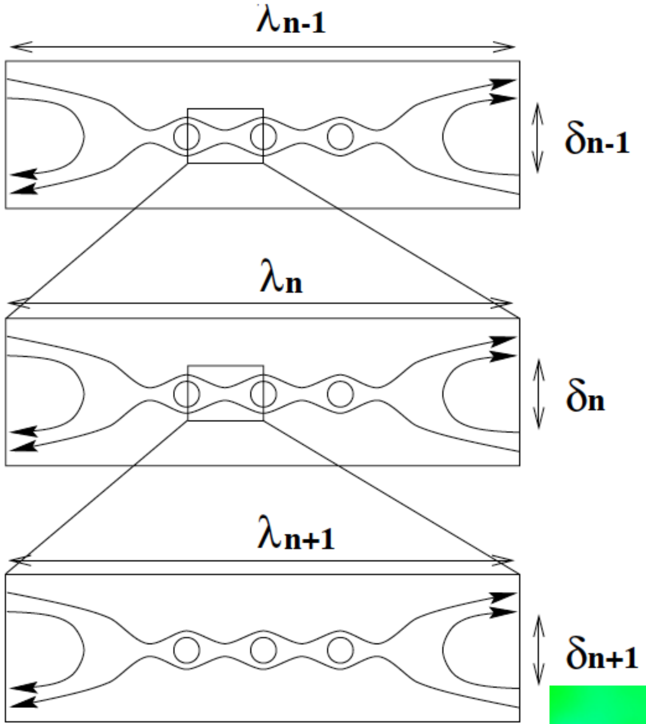


(independently confirmed by Huang et al., PoP '10)

KAUST

Hierarchical Plasmoid Chains

Long current sheets ($S > S_c \sim 10^4$) are violently unstable to multiple plasmoid formation.



(Shibata-Tanuma '01)

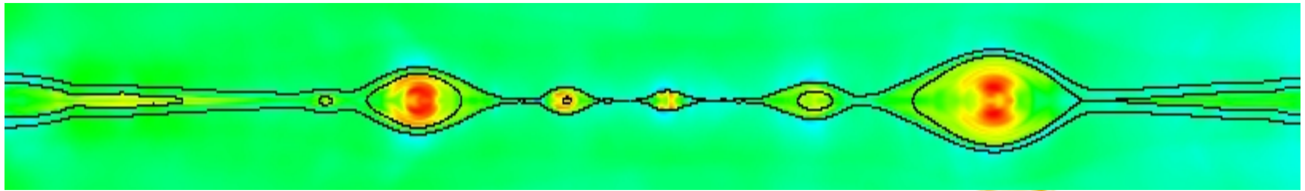
- ***Current layers between any two plasmoids are themselves unstable to the same instability if***

$$S_n = L_n V_A/\eta > S_c$$

- ***Plasmoid hierarchy ends at <u>the critical layer:</u>***

$$L_c = S_c\eta/V_A \quad \delta_c = L_c/\sqrt{S_c}$$

$$cE_c = B_0 V_A/\sqrt{S_c}$$



- ***$N \sim L / L_c$ plasmoids separated by near-critical current sheets.***

Plasmoid-dominated reconnection: ULS model

New theoretical model (ULS) (Uzdensky et al., PRL '10)

Key results:

- Nonlinear statistical steady state exists; ***effective reconnection rate:***

  $$E_{eff} \sim S_c^{-1/2} \sim 0.01 \;\rightarrow\; \text{independent of } S\,!$$

- ***Plasmoid flux and size distribution functions*** are:

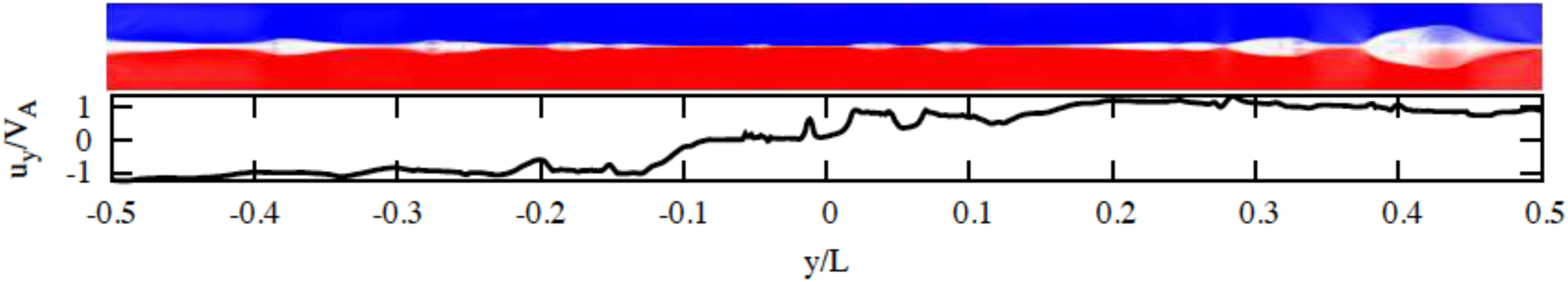
  $$f(\psi) \sim \psi^{-2} \quad ; \quad f(w_x) \sim w_x^{-2} \quad (\text{because } \psi \sim w_x B_0)$$

- ***Monster*** plasmoids form occasionally:

  $w_{max} \sim$ 0.1 L --- can disrupt the chain, observable…

2D Numerical results

Direct numerical simulations to investigate magnetic reconnection at $S>S_c$ and test assumptions and predictions of ULS model



Typical snapshot of B_y. $S=10^6$, domain size: Lx=0.3L; Ly=0.5L; res. 16384^2

Outflows into most plasmoids are approximately Aflvénic

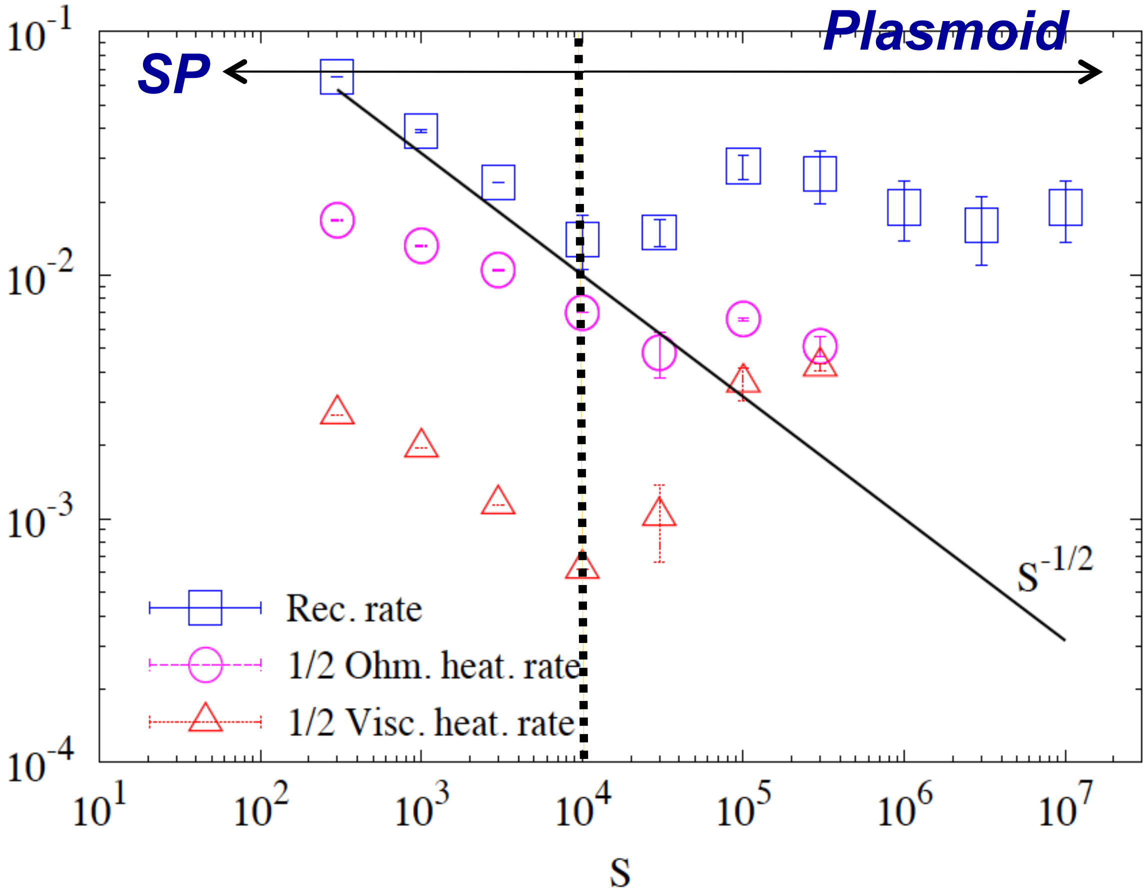
KAUST

Reconnection and dissipation rates

$$\tilde{E}_{\mathrm{eff}} = \left\langle \frac{1}{2L_y V_A} \int dy\, u_x(x = L_x, y) \right\rangle$$

$$Q_\eta = \langle \textstyle\int\int dx dy\, \eta j_z^2(x,y) / 2L_y B_0^2 V_A \rangle$$

$$Q_\nu = \langle \textstyle\int\int dx dy\, \nu \omega_z^2(x,y) / 2L_y V_A^3 \rangle$$

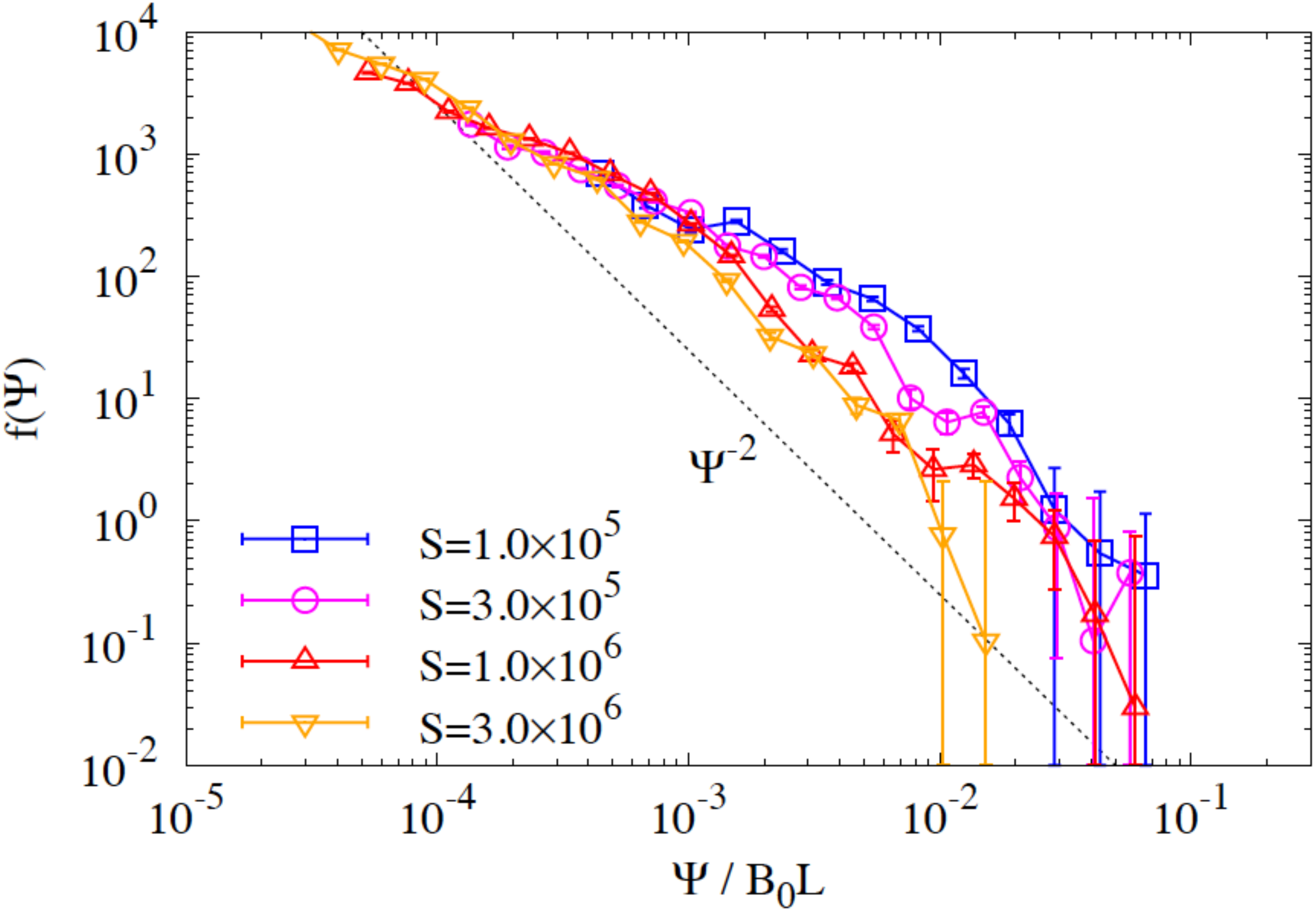


$$\tilde{E}_{\mathrm{eff}} \approx 0.02$$

~ 40% of incoming magnetic energy dissipated into heat

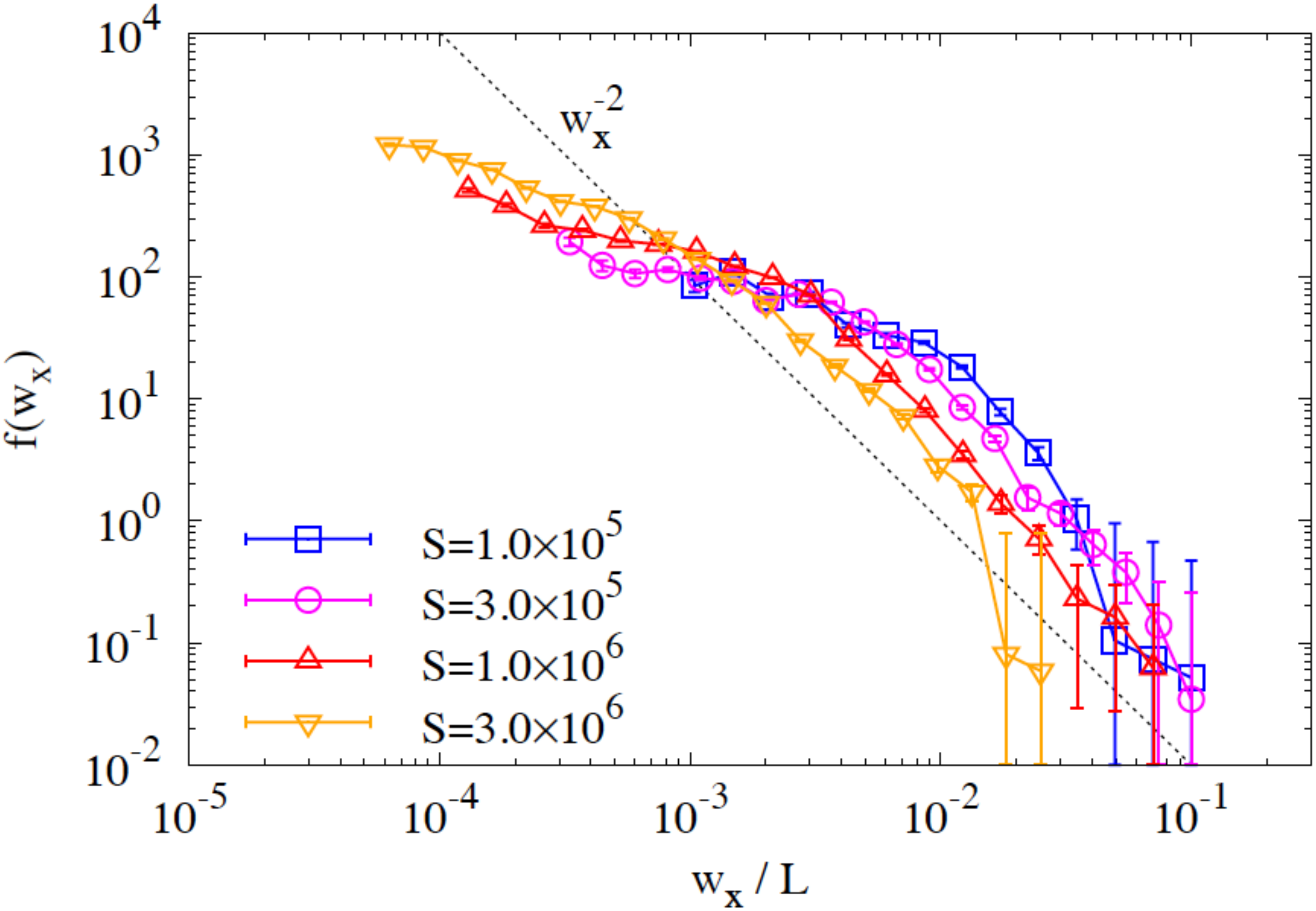
KAUST

Plasmoid flux distribution function



AUST

Plasmoid width distribution function



Width vs. Flux Distribution Function

Diagonal (bold): ULS plasmoids.

There is a significant off-diagonal component

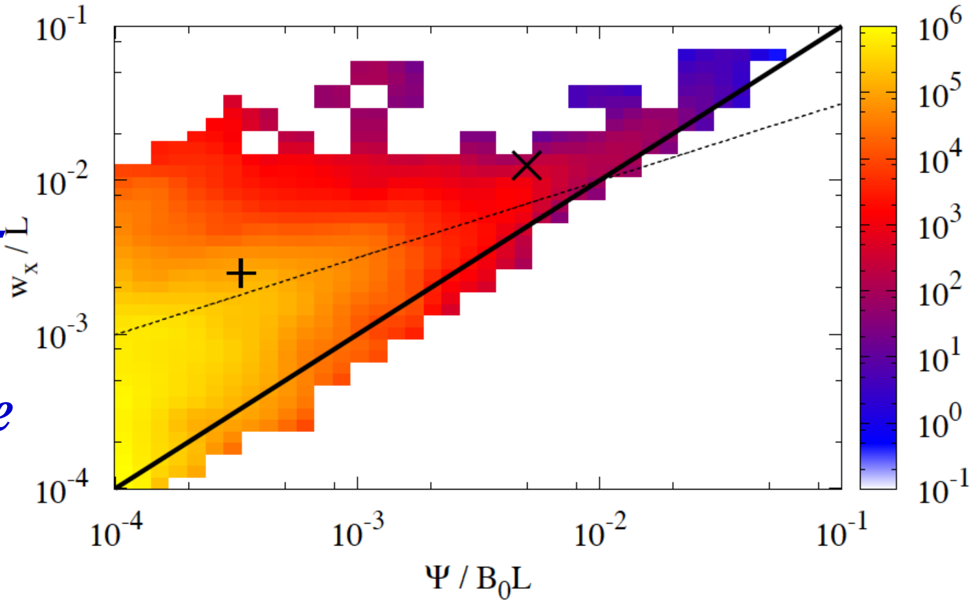
Explained by coalescence between two plasmoids. ULS assumed it to be instantaneous. In reality it takes a finite time.

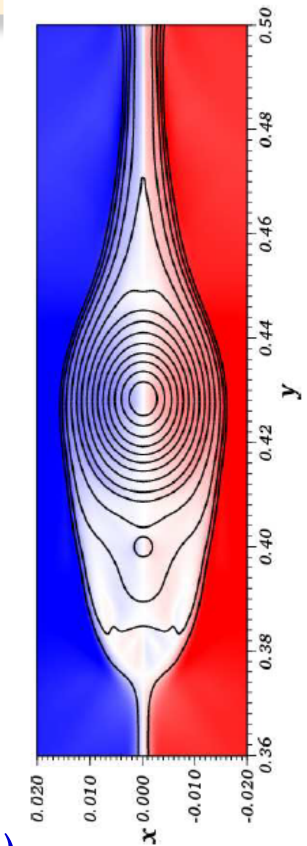
$$B \sim B_0 w_x / w_{x0}$$

$$\Longrightarrow \quad \Psi \sim B_0 w_x^2 / w_{x0}$$

$$w_x < w_{x0} \Longrightarrow w_x > \Psi / B_0$$

i.e., off-diagonal





Width of off-diagonal plasmoids (based on coalescence time arguments) :

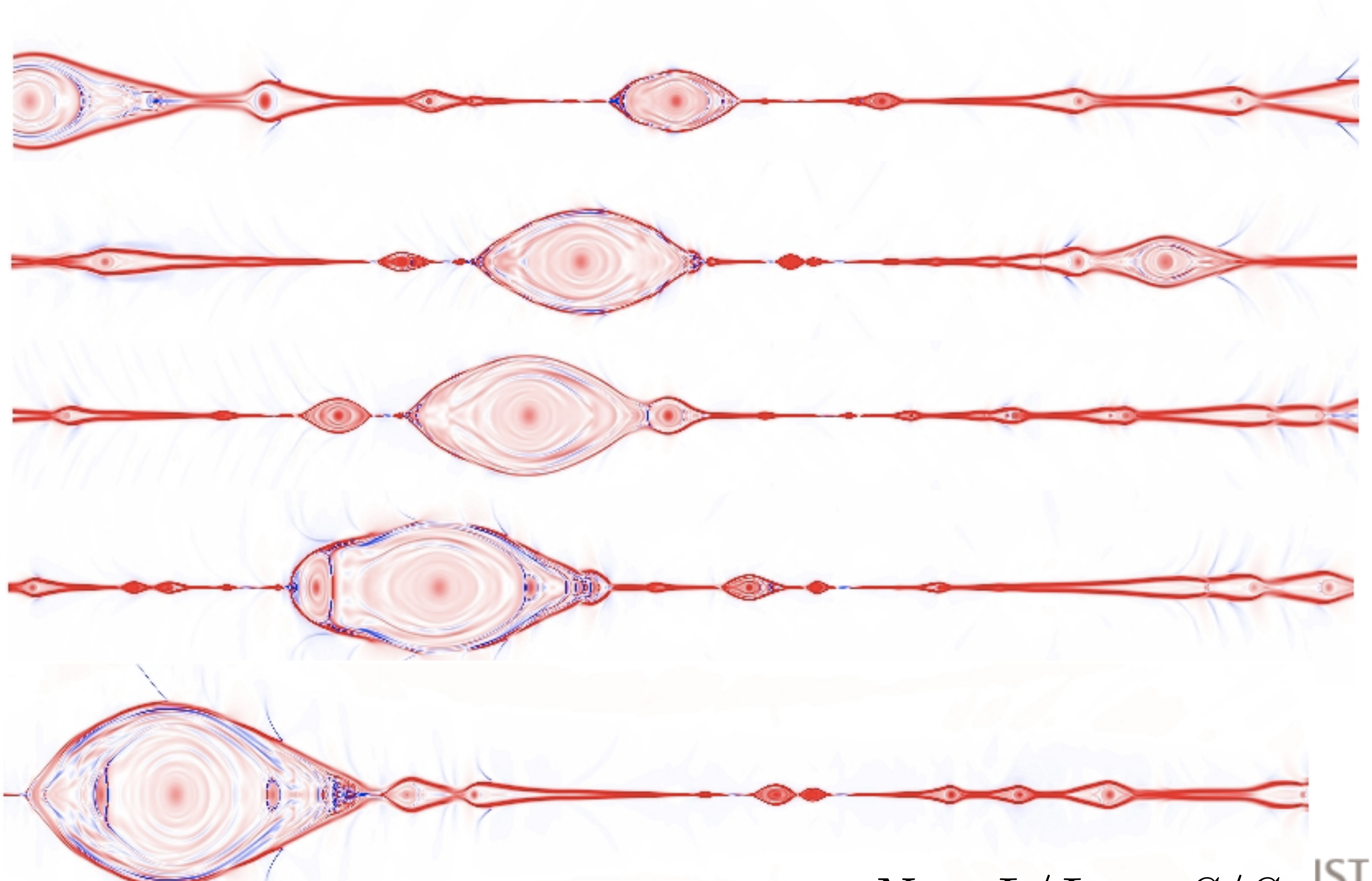
$$w_x/L \lesssim (\Psi/B_0 L)^{1/2} \max(S_c^{-1/4}, S^{-1/6}).$$

$$w_x/L \lesssim \max(S_c^{-1/2}, S^{-1/3}).$$

See Loureiro et al. POP 2012

KAUST

Monster plasmoid formation



Average number of plasmoids in the sheet $N \sim L/L_c \sim S/S_c$

Recent 3D Simulations

Lundquist = Reynolds No.	Guide Field B_z	Domain Size	Mesh Size
$S=Re=10^3$	1.0	(0.3,1,1)	1024x512x128
10^3	0.3	(0.3,1,0.3)	512x256x64
10^3	3.0	(0.3,1,3)	512x256x64
10^4	1.0	(0.3,1,1)	1024x512x128
10^4	0.3	(0.3,1,0.3)	512x256x64
10^4	3.0	(0.3,1,3)	512x256x64
10^5	1.0	(0.3,1,1)	1024x512x128
10^5	0.3	(0.3,1,0.3)	1024x512x128
10^5	0.3	(0.3,1,0.6)	1024x512x256
10^5	3.0	(0.3,1,3)	1024x512x128
10^5	1.0	(0.3,1,1)	1024x512x128
10^5	1.0	(0.3,1,2)	1024x512x256
$3x10^5$	1.0	(0.3,1,1)	1024x512x128
10^6	1.0	(0.3,1,1)	2048x512x128

All runs were performed on the IBM Blue Gene P Shaheen at KAUST on 8192 cores.

KAUST

Motivation for 3D Simulations

- Questions remaining for single fluid resistive reconnection
  - *Does the plasmoid enhanced reconnection rate hold in 3D?*
  - *How do the plasmoid statistics (flux, width distributions) change in 3D?*
  - *What is the effect of the guide field strength?*
  - *What is the morphology of plasmoids in 3D?*
  - *Validity of the ULS model in 3D?*

Resolution Requirements

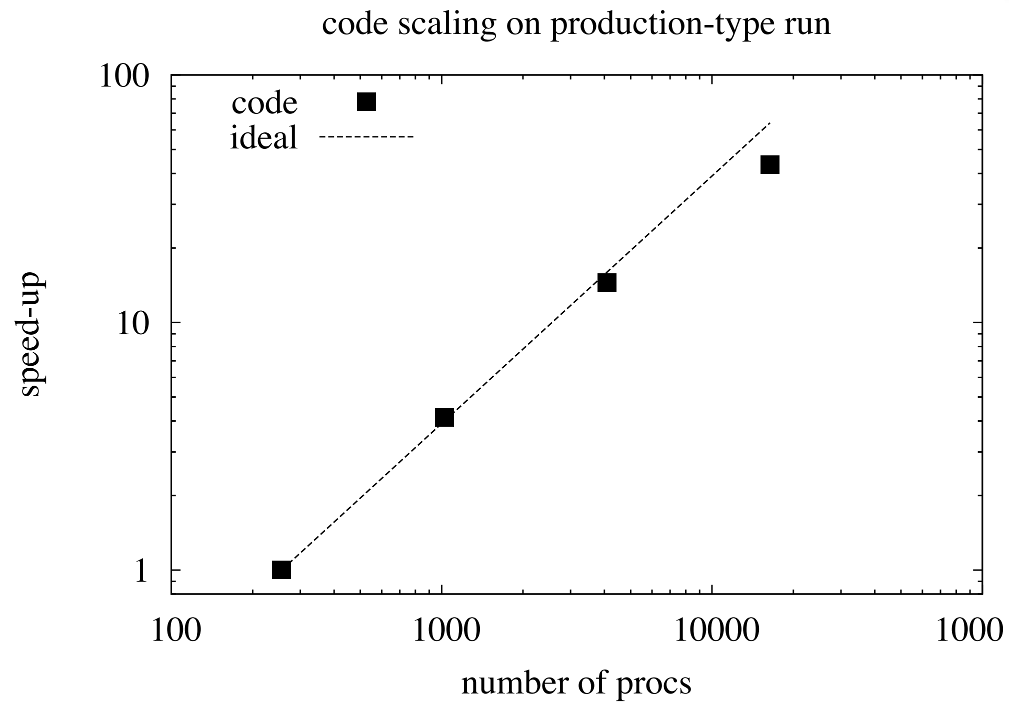
- Current sheet width: $\delta/L \sim S^{-1/2.}$ Relatively "easy" to resolve the thickness of the sheet even at high S number using stretched or adaptive mesh refinement (AMR)
- Recall: 2D linear stability => $k_{max} \sim S^{3/8}$
- Nonlinear simulation indicate average number of plasmoids in sheet $N \sim L/L_c \sim S/S_c$
  - *$S=10^6 \sim O(10^2)$ plasmoids ➔ $>O(10^3)$ points along the current sheet*
  - *$S=10^7$ ➔ $\sim O(10^4)$ points. Nearly impossible today?*
- Further recall: plasmoid chains depict a hierarchical structure. This implies that even with AMR this is an extremely challenging computational problem

KAUST

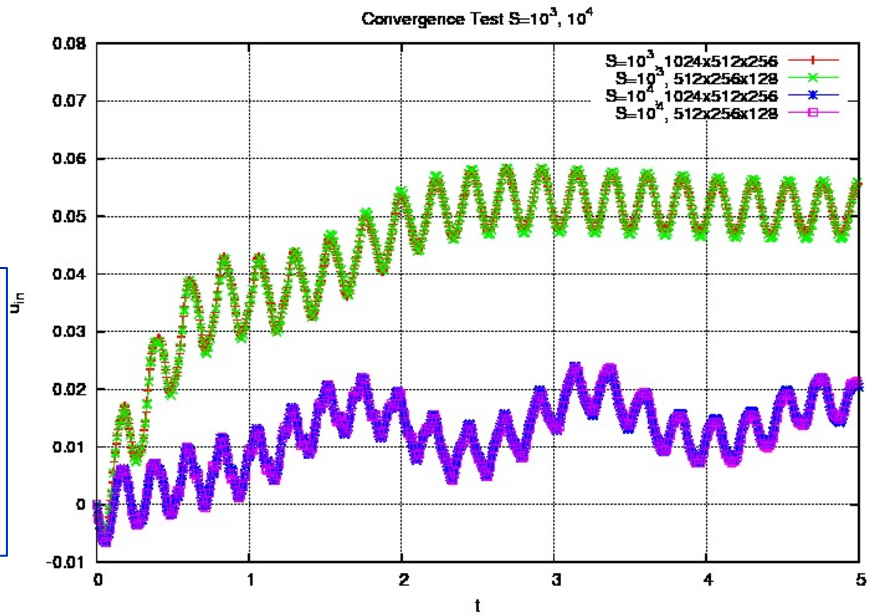
Numerical Method

- Finite volume approach to solve resistive MHD equations
- Operator splitting with second order TVD Runge-Kutta for hyperbolic portion of the MHD equations
- Upwind methods with linearized Riemann solvers for MHD to compute hyperbolic fluxes
- Implicit treatment of diffusion terms
  - *Multigrid solver for diffusion*
  - *Converges in 1 V-cycle with residuals $< 10^{-12}$*
- div(B)=0 constraint enforced by evolving auxiliary equations for the vector potential A where B=curl(A)

Code Scalability/ Convergence Test

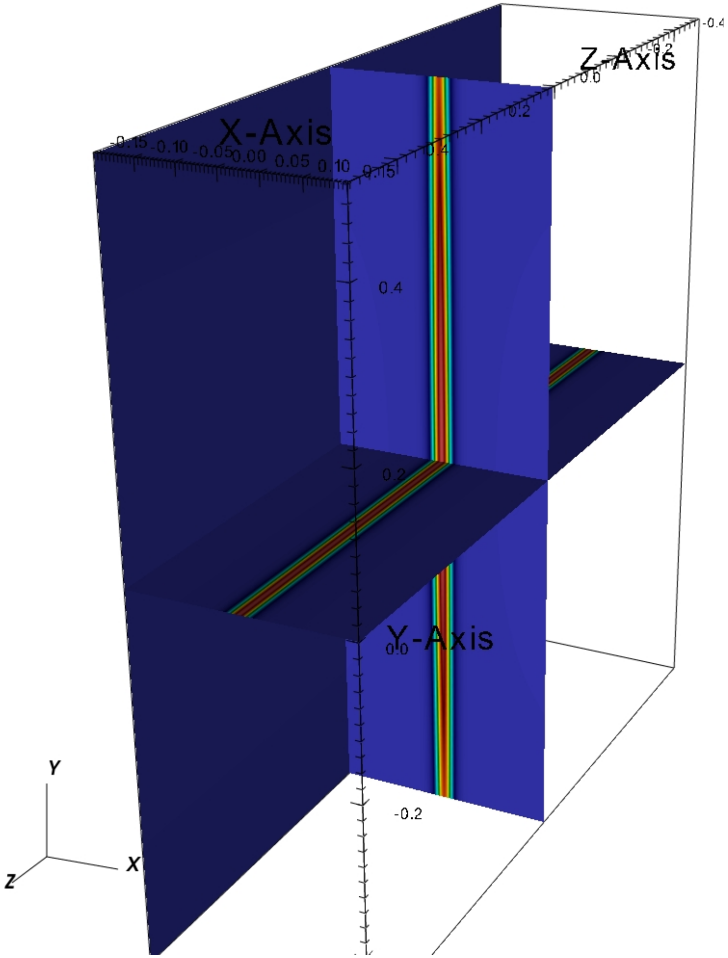


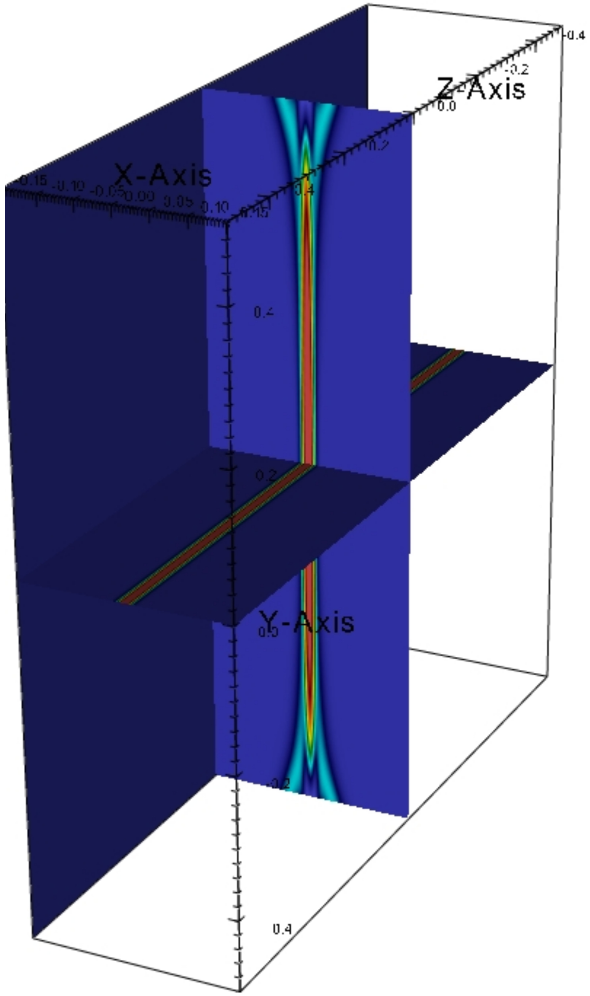
Strong scaling in 2D (4096^2 mesh) up to 16K cores



Convergence Test in 3D Instantaneous Reconnection Rate for S=10^3, 10^4 shows convergent results

Simulation Results: S=10^4

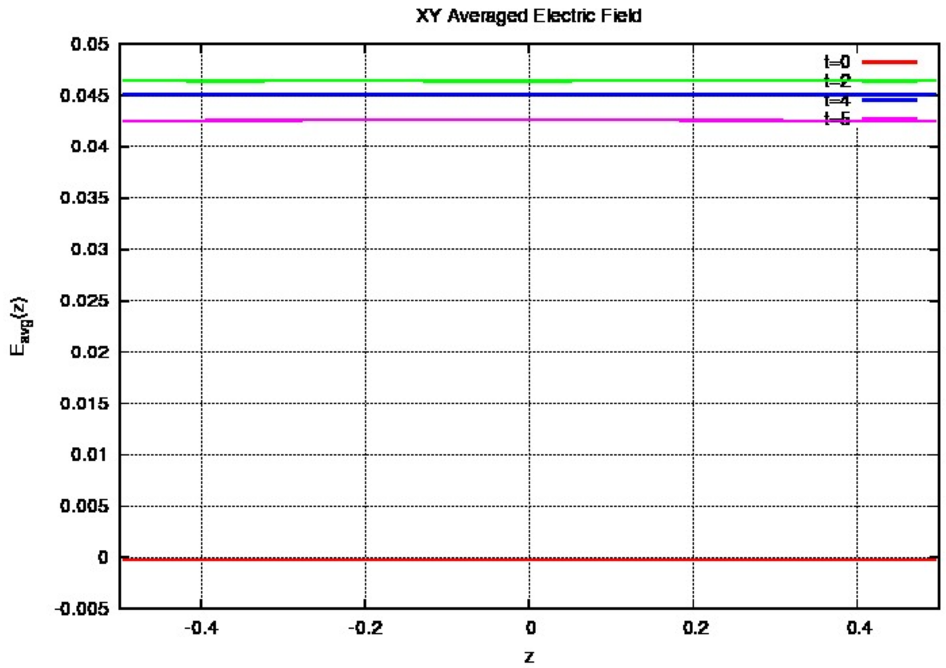


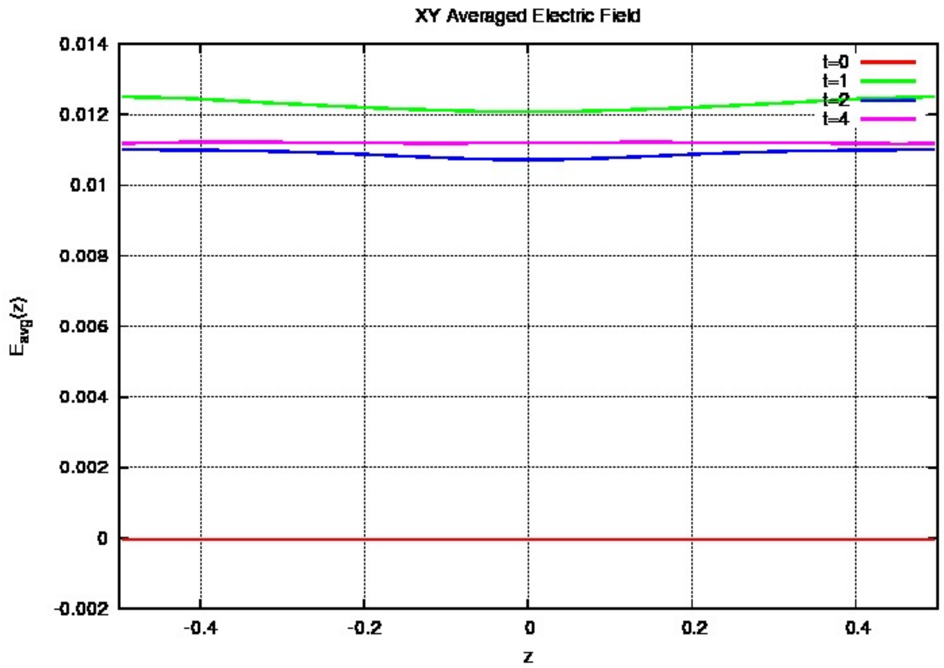


t=0

t=7.3

Plane Averaged Electric Field





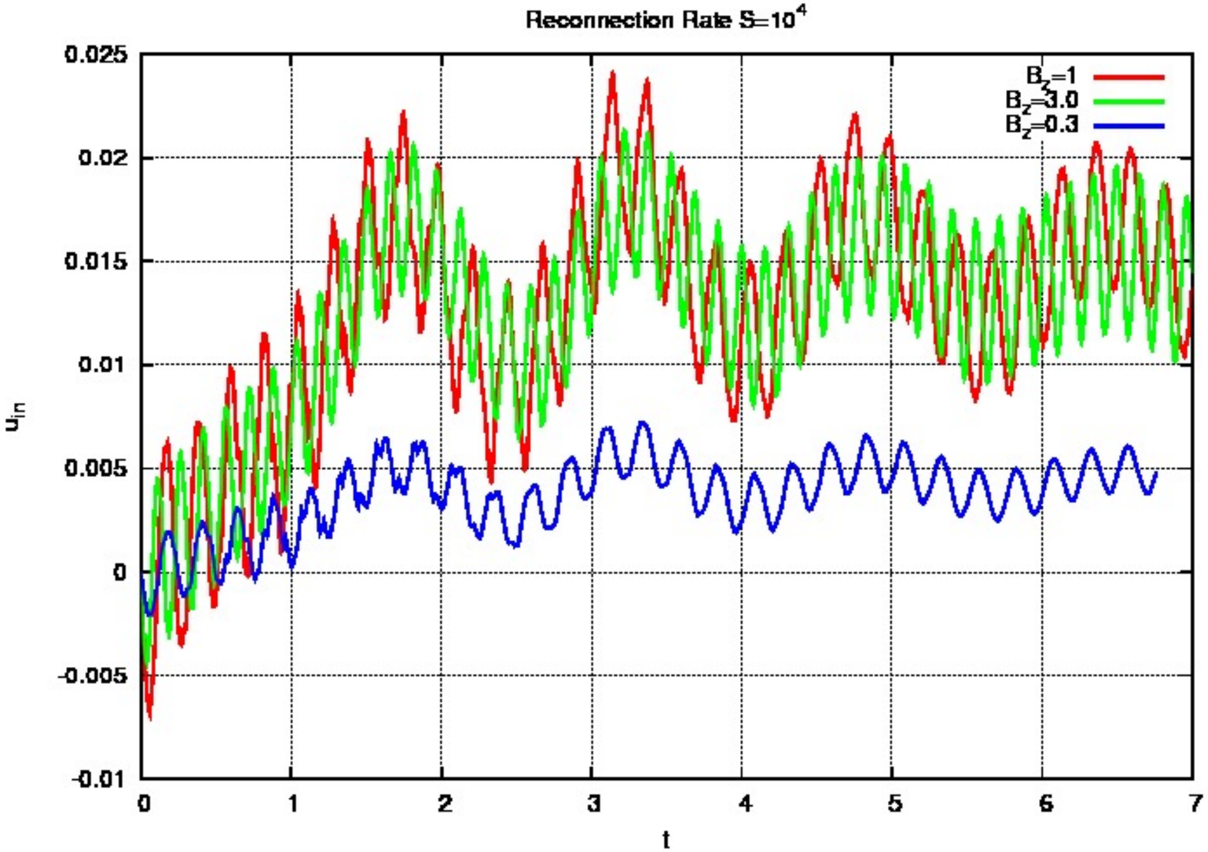
$S=10^3$

$S=10^4$

Reconnection is quasi2D for low S proceeding in a SP-like fashion

KAUST

Dependence on B_z

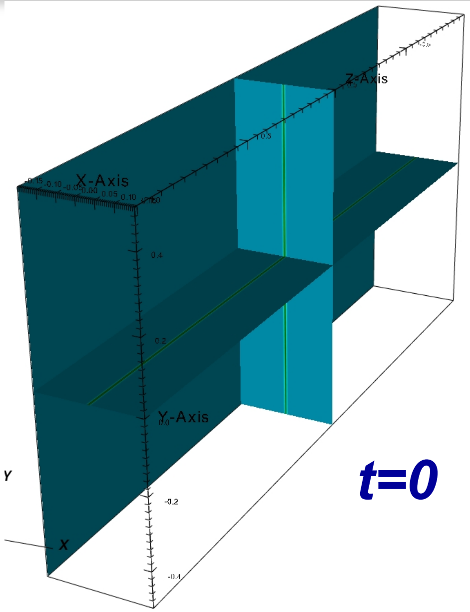


S	B_z	Avg. Recon Rate x 10^2
10^3	0.3	1.55
10^3	1.0	4.93
10^4	0.3	0.47
10^4	1.0	1.43 (≈2D)
10^4	3.0	1.47 (≈2D)

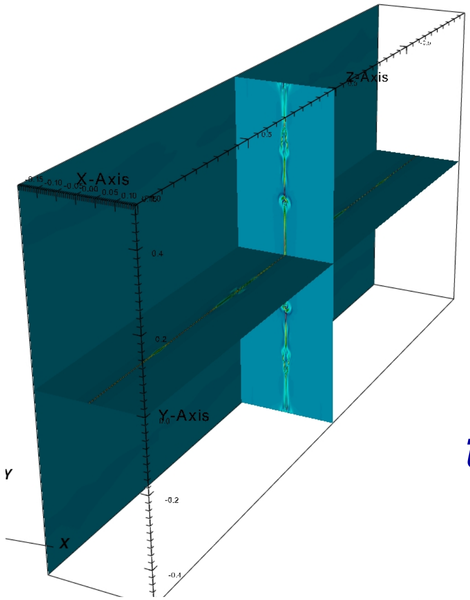
Reconnection rate becomes independent of B_z for large B_z

KAUST

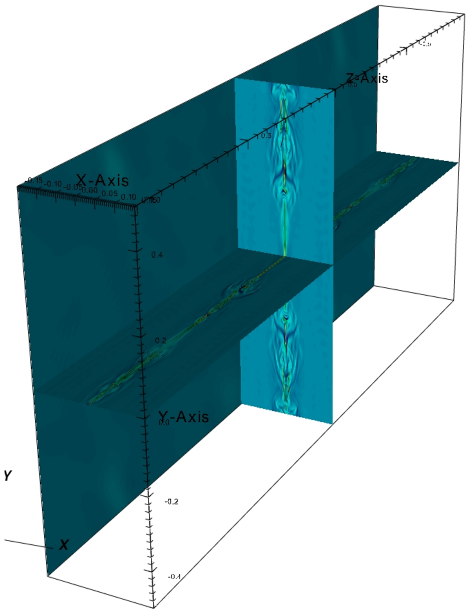
Current Sheet Evolution: $S=10^5, B_z=1$



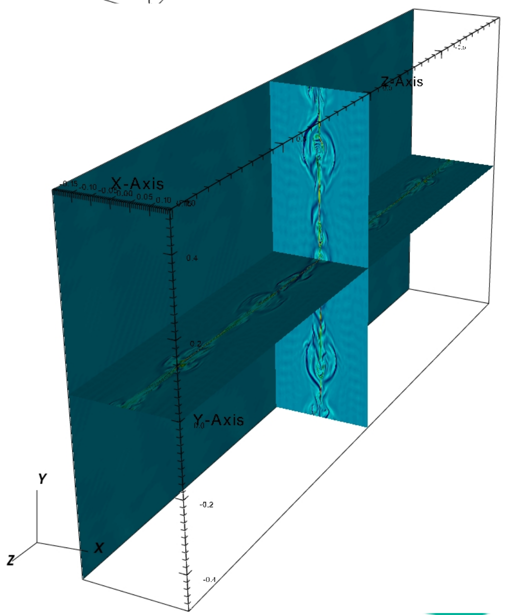
t=0



t=3

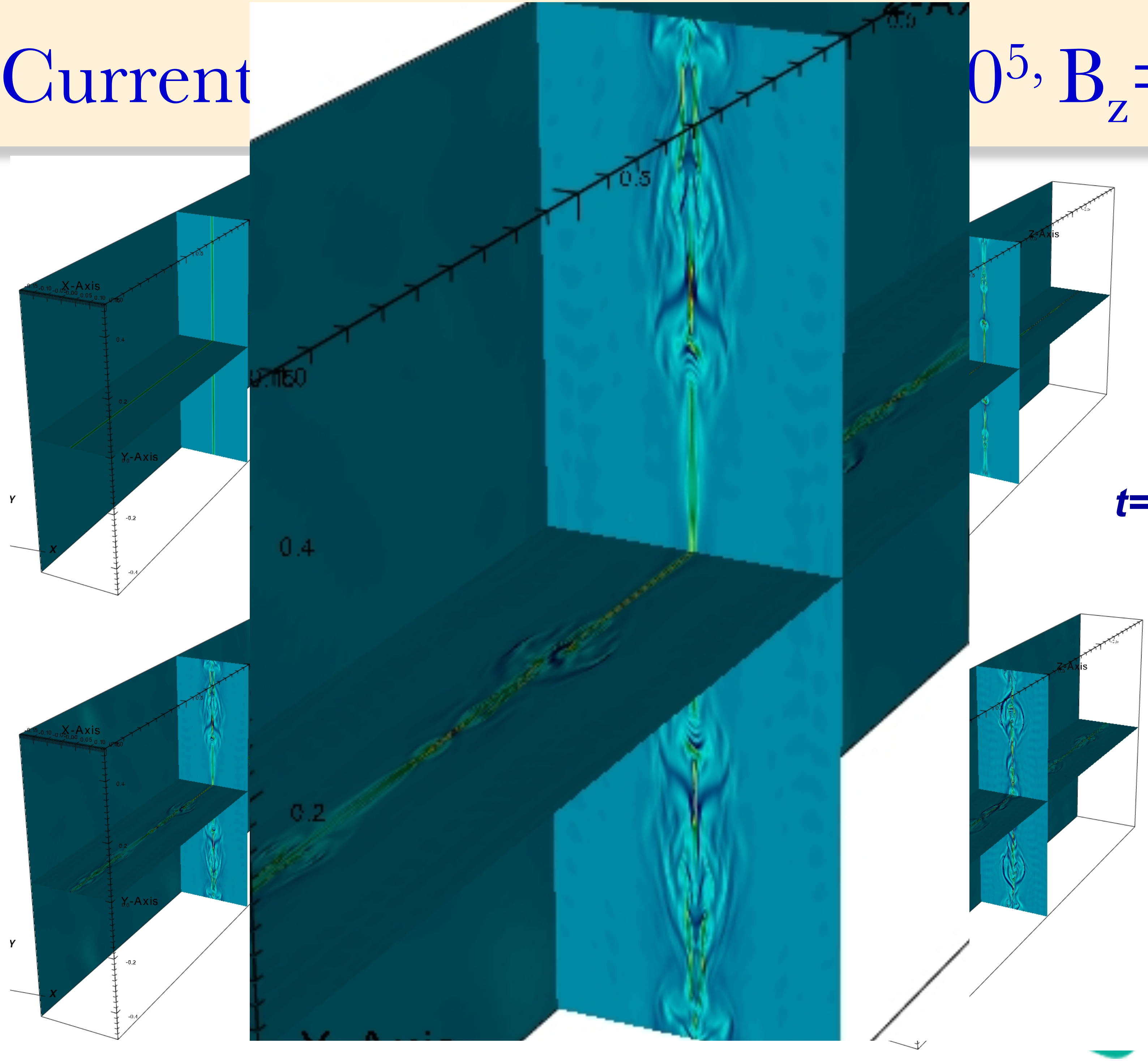


t=4



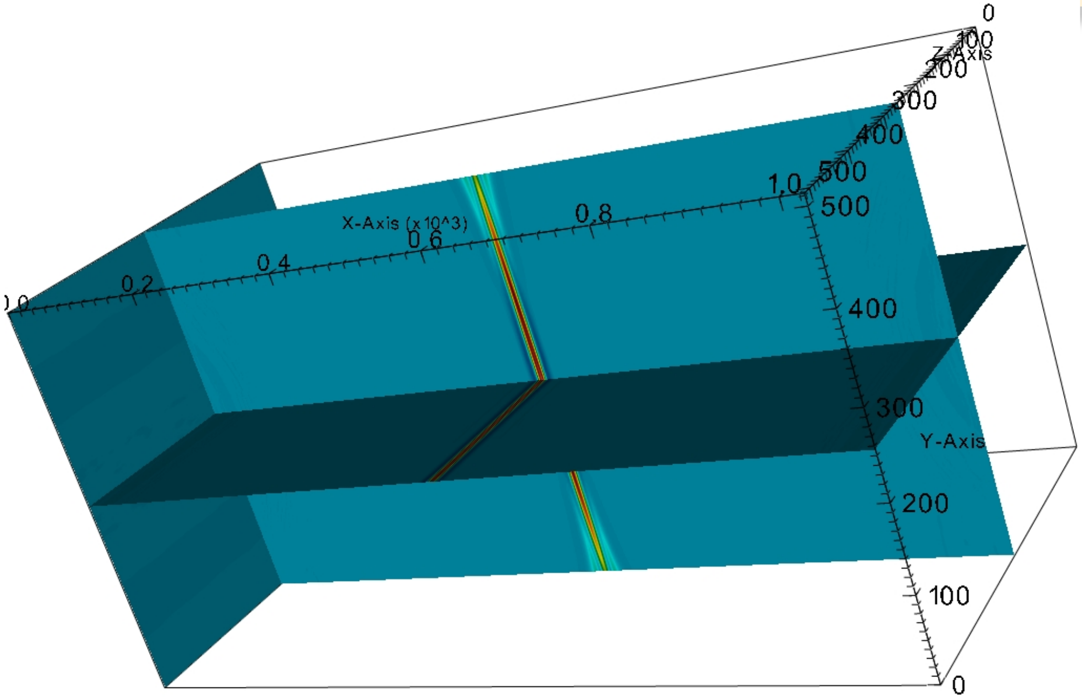
t=5

Current ... 0^{5}, $B_z=1$

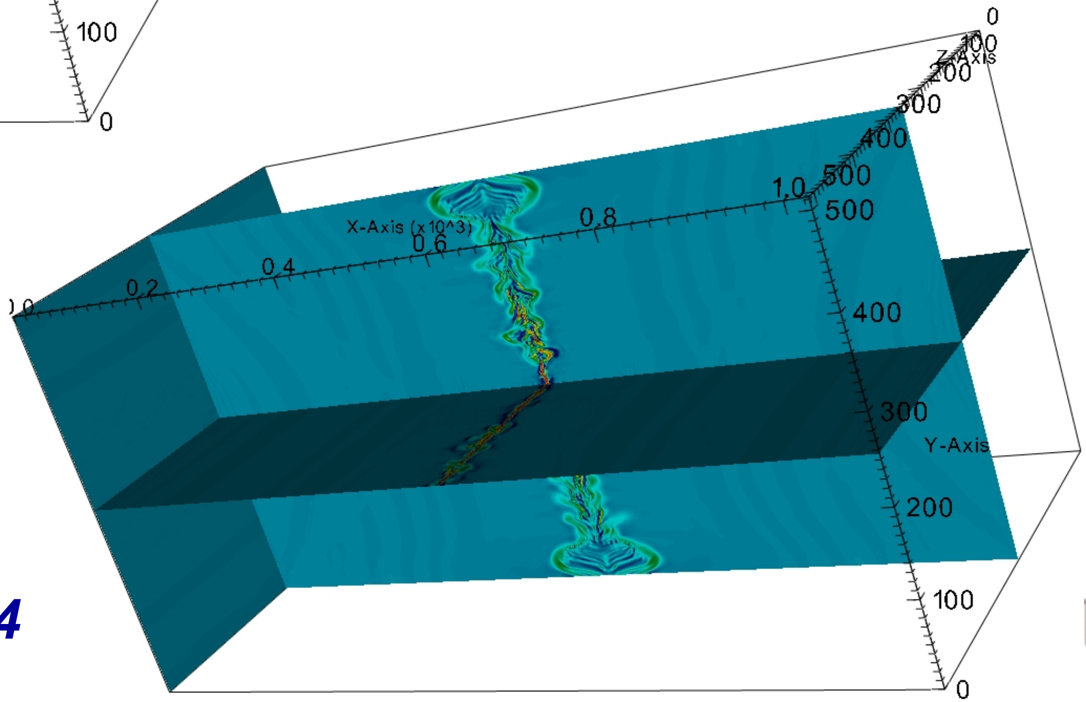


KAUST

Current Sheet Evolution: $S=10^6$, $B_z=1$

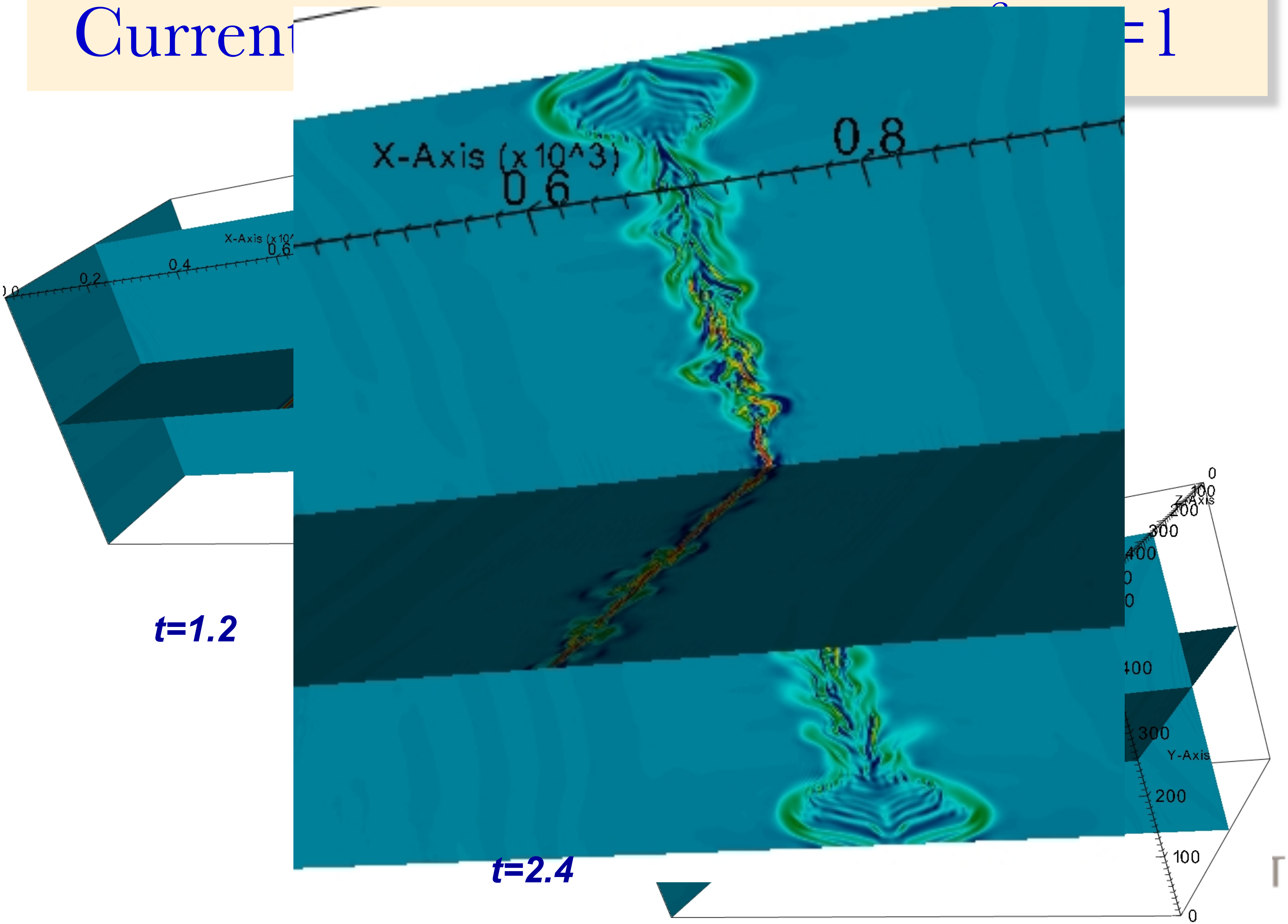


t=1.2

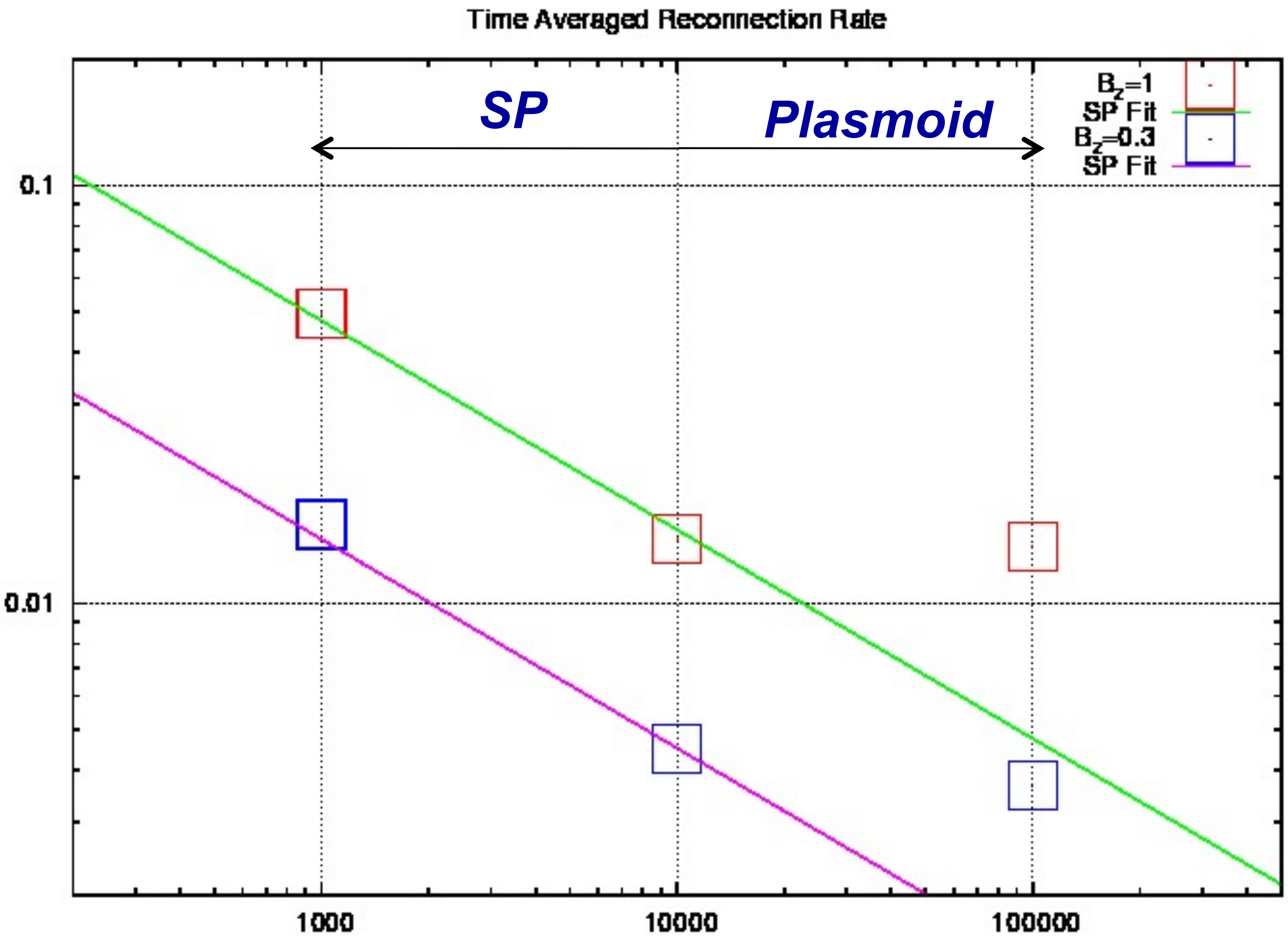


t=2.4

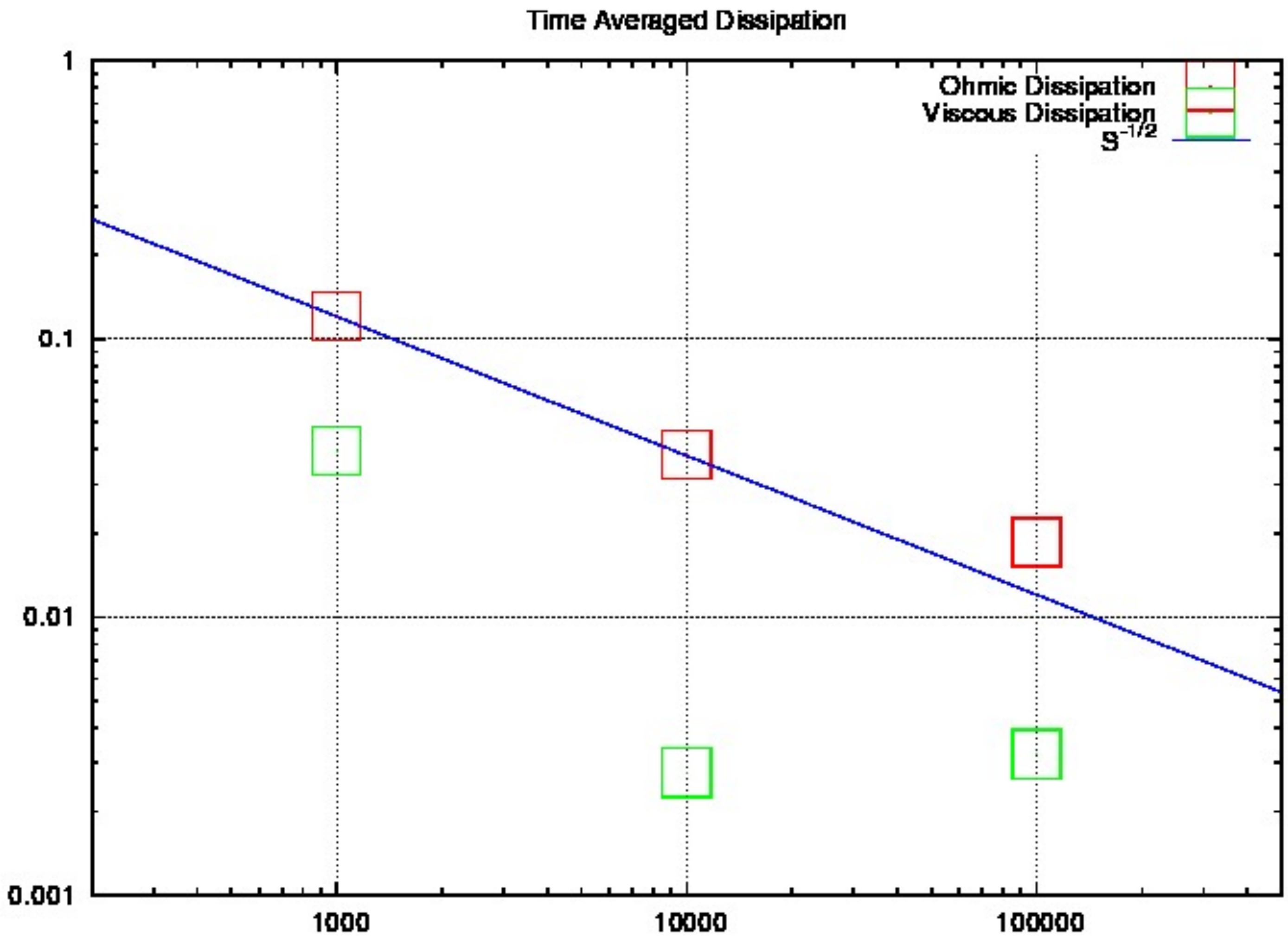
Current ... =1



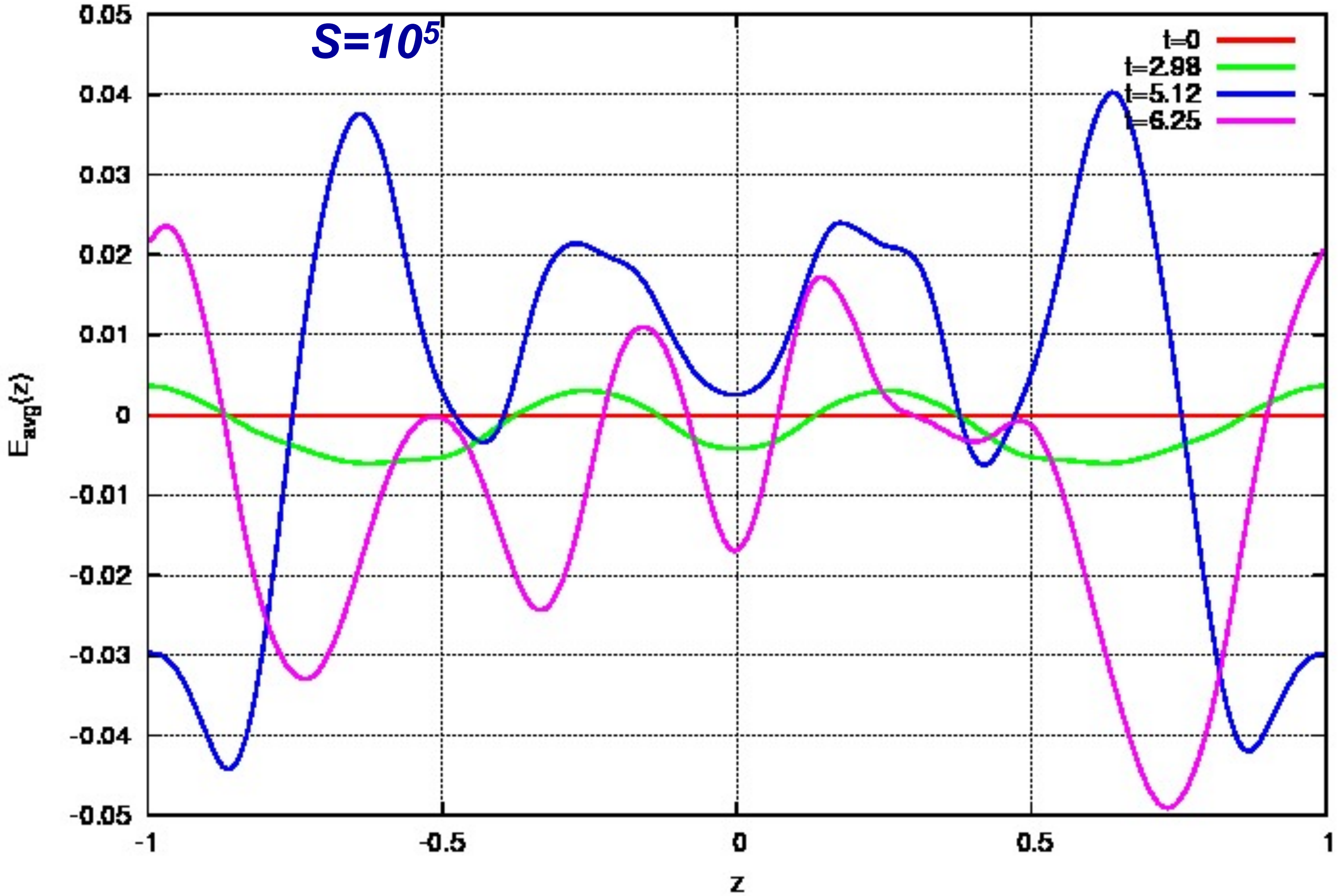
Time Averaged Reconnection Rate



Average Dissipation



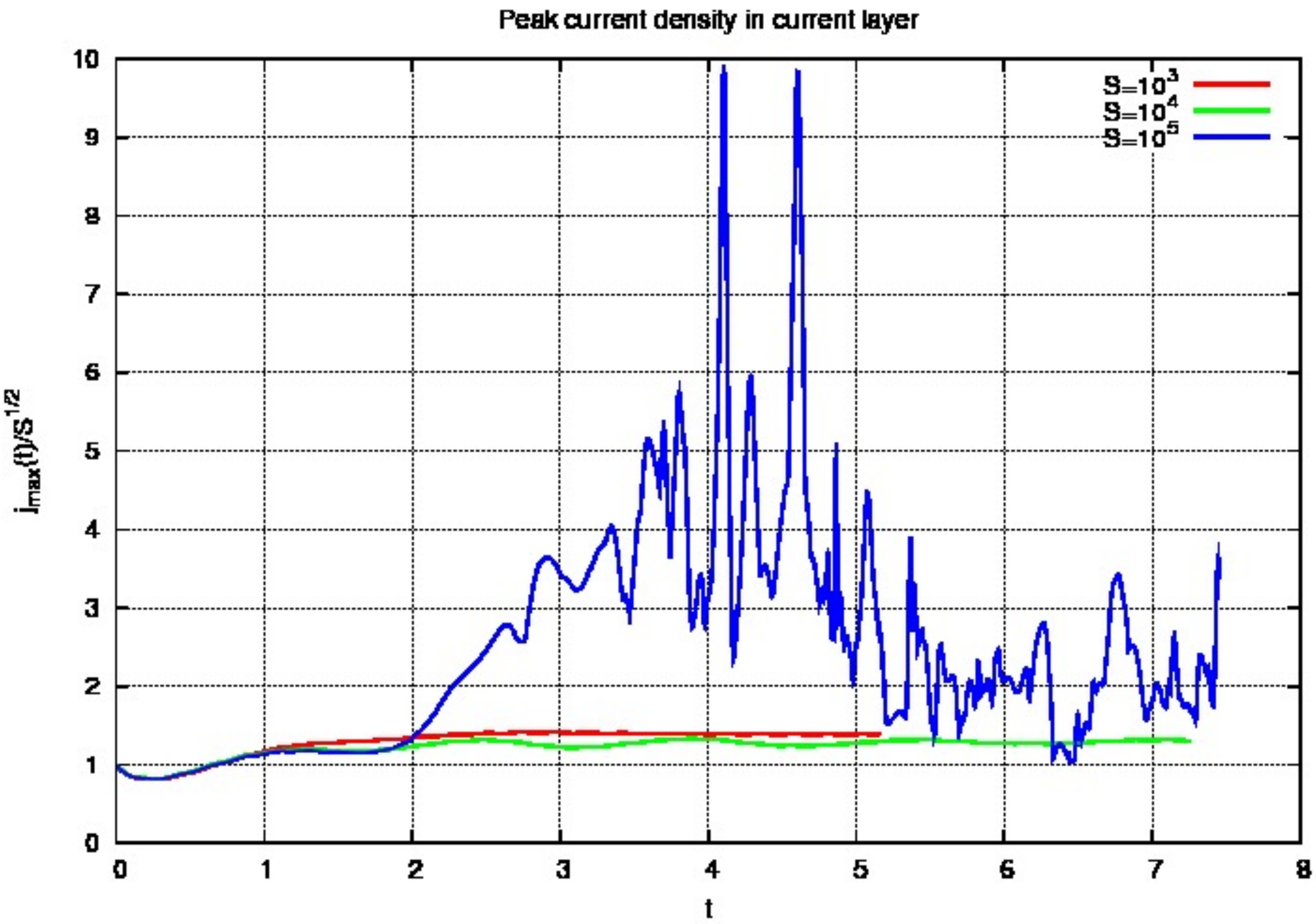
XY Plane-Averaged Electric Field



Strong variation of xy-plane averaged Electric field

KAUST

Time Evolution of Peak Current Density



KAUST

Conclusions

- Current sheets predicted by the SP theory are ***violently unstable*** to the formation of ***high wavenumber plasmoid chains*** (Loureiro *'07*, Samtaney '09*)*.

- ***MHD reconnection in large S systems is fast, dynamic, bursty.*** Sweet-Parker theory inadequate. Plasmoid-dominated reconnection requires ***statistical description***. The *ULS model* (Uzdensky *'10) is the first attempt at quantifying plasmoid mediated reconnection*

- Numerical simulations in 2D confirm and partially amend the theoretical predictions of the *ULS* model

- ***Both the reconnection and the dissipation rates asymptote to S-independent values.*** ~ 40% of incoming energy dissipated into heat.

- Preliminary 3D simulations indicate that plasmoid-dominated reconnection rate is significantly larger than SP for $S=10^5$
  - *Even larger resolution required to provide conclusive evidence for $S=10^6$ and beyond that reconnection rates asymptote to S-independent values*
  - *$S=10^3$, 10^4 reconnection is quasi2D with variations in z rapidly decaying away*
  - *Smaller guide field has a lower reconnection rate than larger values of B_z but reconnection rates (at least for lower S) become independent of B_z for $B_z>1$*

KAUST

Acknowledgement

- KAUST Research Funds for R. Samtaney

- KAUST Supercomputing Laboratory for computer time on the IBM Blue Gene P (Shaheen) ~ 30 million core hours.