Effects of graviy and the state of pore-fluid pressure in the bedrock surrounding an idealised magma chamber, compared to the Mogi approach - Muriel Gerbault, Frederic Cappa, Riad Hassani

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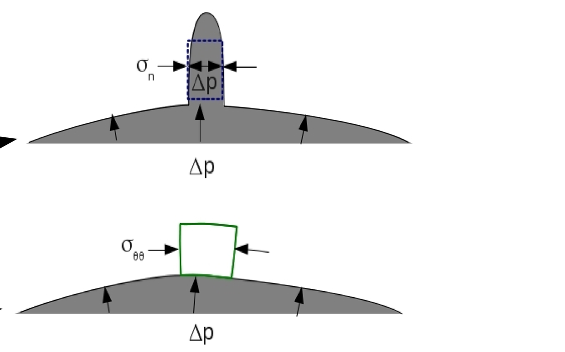
1- INTRODUCTIVE SUMMARY

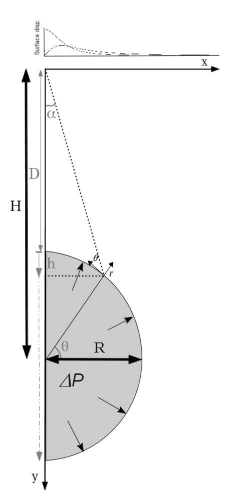
Predictions of surface displacements above an inflating chamber generally assume that magma overpressure is limited by the bedrock tensile strength. This results from the assumption that the same failure criterion rules the top surface and the chamber's wall (*Tait et al., 1989*), thus neglecting the effect of gravity. This assumption is valid if one at least of the two conditions below are satisfied:
- a well-oriented fluid-filled fracture already exists (Rubin, 1995),
- the bedrock is at a state of near lithostatic pore-fluid pressure.

Our study addresses the situation in which neither of these conditions are fulfilled. If we consider the stress balance in a relatively intact bedrock adjacent to a spherical or infinitely long cylinder, the gravity body force actually resists tensile failure, thus leading to a much larger pressure threshold for failure (*Grosfils, 2007*). We show here analytically and numerically that:
- shear -failure occurs instead of tensile failure (*Gerbault, 2012*).
- the state of pore-fluid pressure in the bedrock actually controls this process (*Gerbault et al., 2012*).

We compare elasto-plastic solutions of surface displacements and patterns of failure in plane-strain at fixed internal overpressure with three different numerical codes.

We propose to explain paradoxes often mentioned in the literature :
1) high internal overpressures are required to fit theoretical and observed top surface deformation above volcanoes is /not/ realistic, but simply means that the bedrock is not at a state of lithostatic pore-fluid pressure (hydrostatic is more common, e.g. *Zoback, 2007*),
2) the initiation of dike or sills (e.g. mode I features) are associated to shearing mechanisms (double couple focal mechanisms), rather consistent with a mode II failure. We argue that shear failure initiates indeed, which would then propagate in mode I, once filled with fluids.





CONVENTIONAL ELASTIC PREDICTIONS - A pressurized cavity in an infinite elastic medium exerts from the wall radial (σ_R) and tangential (σ_T) stresses . The tangential stress is function of the normal stress. In 3D, σ_T=- σ_R/2, and in 2D, σ_T=- σ_R(*Timoshenko & Goodier, 1951*).

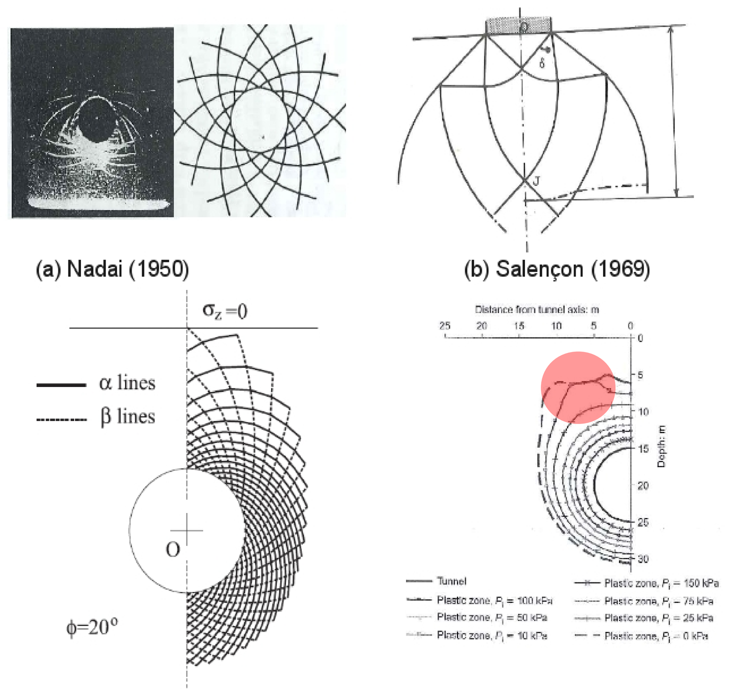
Without accounting for gravity, tensile failure is predicted (*Jeffery ,1920*) :
- at the top surface when $\Delta P_S = T(H^2-R^2)/2R$,
- at the chamber wall when $\Delta P_T = T(H^2-R^2)/H^2$.

Vertical and horizontal displacements at the surface can be evaluated, by accounting for the free-surface effect (factor $C=1+2\tan^2\alpha$). In 3D and for a Poisson's ratio ν= 0.25, this is the famous approximation (*Mogi 1958, McTigue 1987*):

$$u_y = \frac{3\ \Delta P}{4\ G} \cdot \frac{R^3 \cdot H}{(x^2+H^2)^{3/2}}.$$

FAILURE PREDICTED IN ENGINEERING PLASTICITY

The problem compares well with common engineering situations such as flat/circular metal indentation (a,b), and tunnelling (c,d).



(a) Nadai (1950) (b) Salençon (1969)

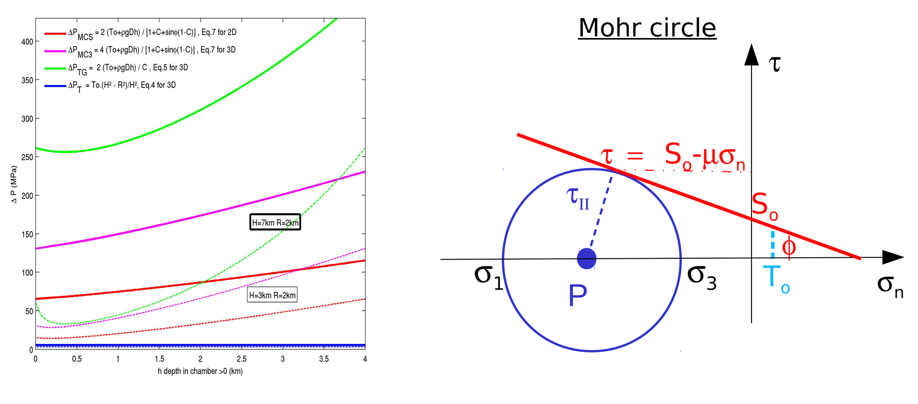
(c) d'Escatha & Mandel (1974) (d) Massinas & Sakellariou(2009)

2 - ANALYTICAL REASONNING: PRESSURE THRESHOLD & SHEAR vs. TENSILE FAILURE

1) CONVENTIONAL SOLUTION Most authors suppose that overpressure ΔP is limited by the tensile strength of the bedrock above the chamber: $\Delta P_T < \sigma_3 + T_o$, σ_3 is non-zero only if tectonic stress is present.

2) ACCOUNTING FOR GRAVITY, *Grosfils (2007)* shows that the tangential (or hoop) stress $\sigma_{\theta\theta}$ // to the chamber wall is rather : $\Delta P_{TG} = 2(T_o + \rho g(D+h))/C$.

3) MOHR-COULOMB failure criterion provides : $\Delta P_{MC} = \sin\phi.(T_o + \rho gh)$, for C=1.

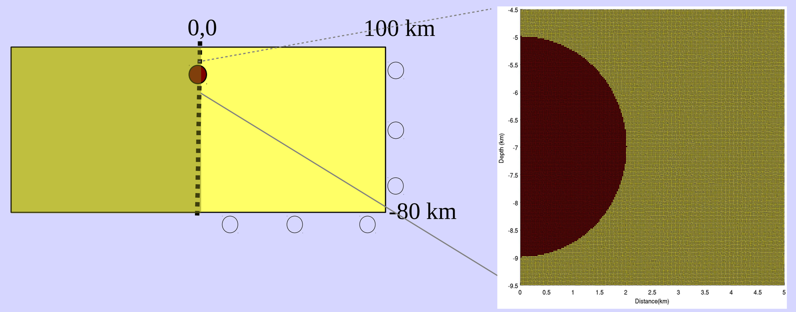


Tensile yield $\Delta P_{TG} > \Delta P_{MC}$ shear yield **FAILURE IN SHEAR !**

3 -NUMERICAL METHOD and SETUP

Three numerical codes are used to simulate elasto-plastic deformation resulting from an increase in uniform internal pressure, in order to gain confidence on the quality of the results. We address the sensitivity of failure initiation to the initial pressure conditions, the mesh geometry, and the hydromechanical coupling between fluid flow and deformation.
- The first code used is **Parovoz** (*Poliakov & Podladchikov, 1992*) based on the FLAC FD method (*Cundall & Board, 1988*).
- The second code is **FLAC3D** (*Itasca Consulting Group, 2006*), which was designed to simulate geomechanical problems. FLAC3D builds radial meshes and incorporates the coupling between fluid flow and deformation, either in static and dynamic modes.
- The third code is **Adeli** (*Hassani et al., 1997*), a FEM code based on the dynamical relaxation method dedicated to geodynamics.

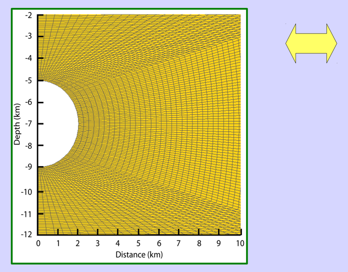
Chamber centre H =7 km & radius R = 2 km



In **Parovoz** the mesh is built with quadrilaterals, and the chamber is meshed in order to achieve high resolution (20 m).

Boundary conditions :
Half of the problem is modeled due to vertical symmetry.
Top surface is stress free. Left, bottom, right borders free-slip.
Internal pressure (ΔP) increases progressively inside the chamber.

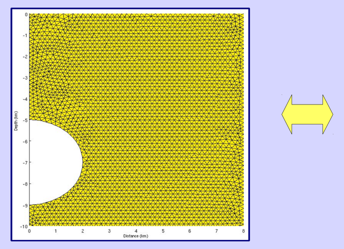
Rheologies:
* Bedrock domain is elasto-plastic: $\lambda = \mu$ = 20 Gpa.
Mohr-Coulomb friction and cohesion Φ=30°, So =10 MPa.
Tensile strength is To=So/tanΦ or set to a cutoff T=5 Mpa.
Pore-fluid pressure acts in the yield criterion with $p_f = \lambda.\rho gz$.
* Magma chamber is elastic: $\lambda = \mu$ = 2 GPa.



FLAC3D is a commercial geotechnical software (www.itascacg.com). It allows to build a radial mesh in plane-strain (resolution 50m) from polyhedral elements. Here, loading on the chamber wall is applied instantaneously, and therefore the associated fluid pressure and deformation are mainly undrained. Consequently, the change in fluid pressure (ΔP_f) is due to the change in the ratio of volumetric strain (or dilatation) and the initial rock porosity (n_0) :

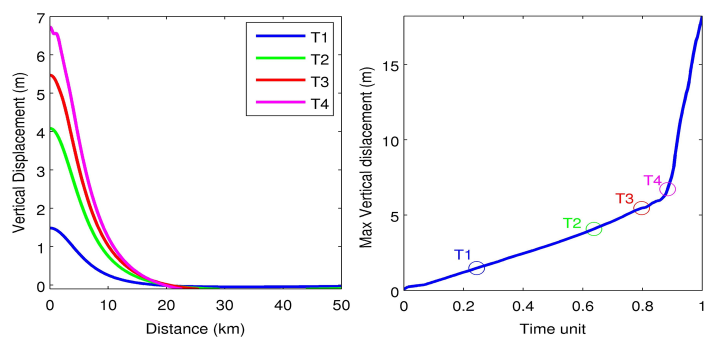
$$\Delta P_f = -K_{f\Delta} (\Delta V/V) ,$$

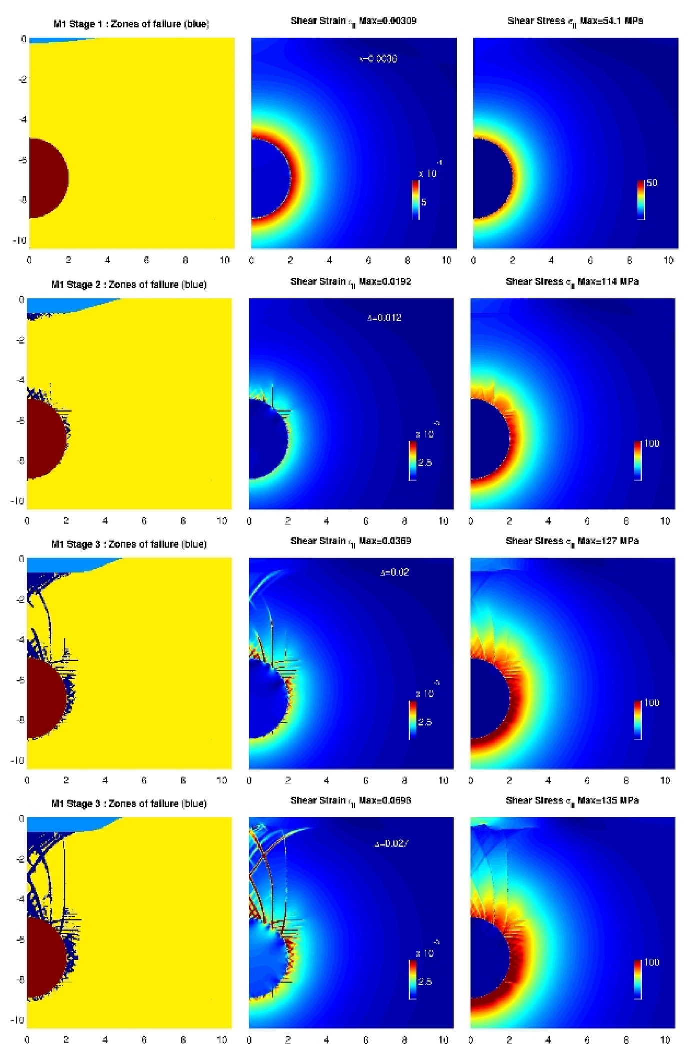
where K_f is the fluid bulk modulus (K_f = 2 GPa, and fluid compressibility $C_f = 1/K_f$), V is the initial pore volume (V = n_0 in a unit volume of rock, if the pore spaces are fully saturated with fluid as is the case in our study), and ΔV is the volume change due to deformation.



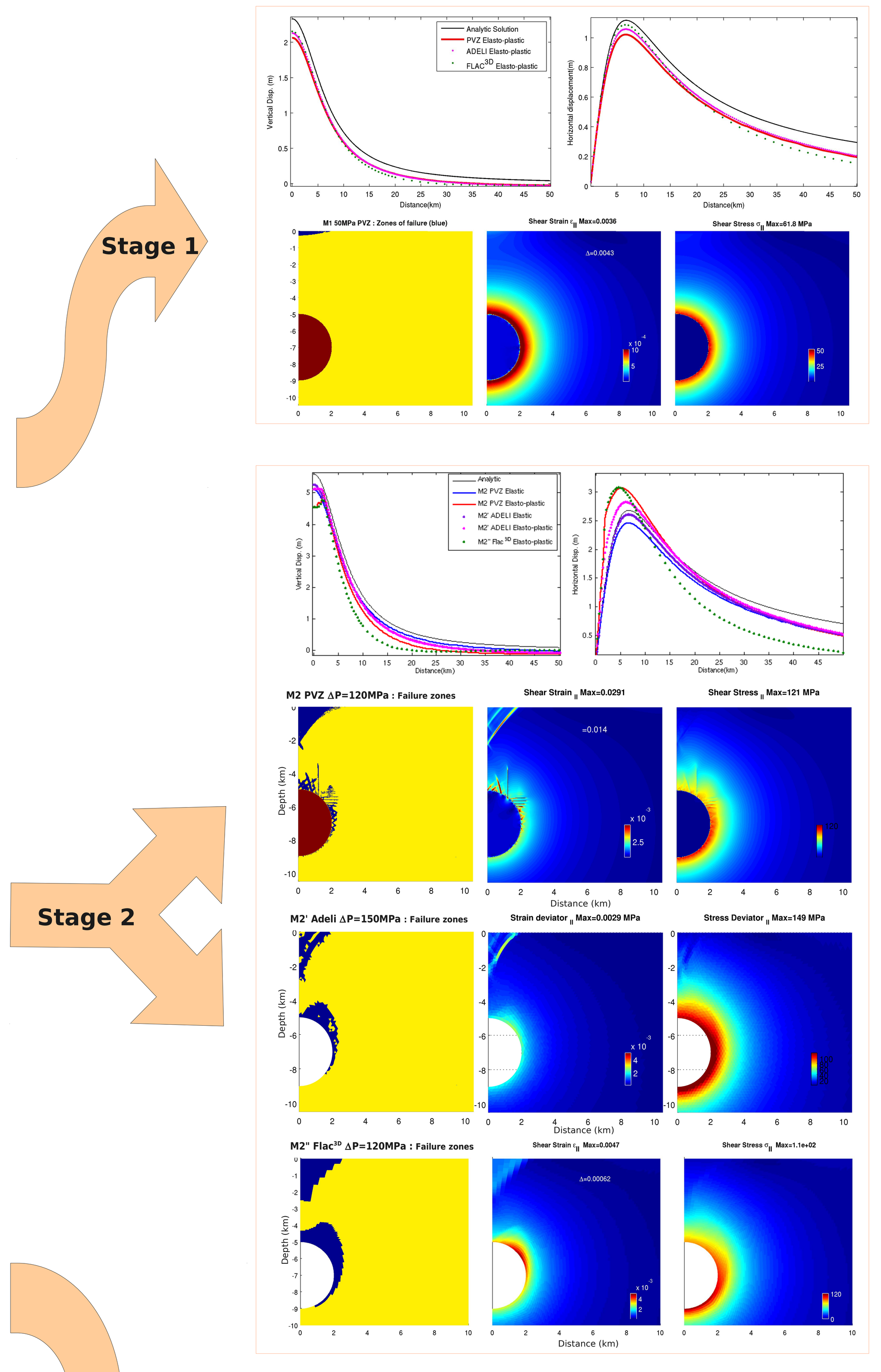
ADELI2D (e.g. *Hassani et al., 1997; Hassani & Chery, 2001, Bonnardot et l., 2008*) allows to build a radial mesh of triangles. ADELI uses the Mohr-Coulomb friction and cohesion parameters in a Drucker-Pareger based plastic formulation.
Download site and further details at http://www.dstu.univ-montp2.fr/PERSO/chery/Adeli_web/.
ADELI3D exists and models have been ran : 2 months time for coarse resolution...

4 - FAILURE INITIATION AND PROPAGATION NO FLUIDS IN THE BEDROCK





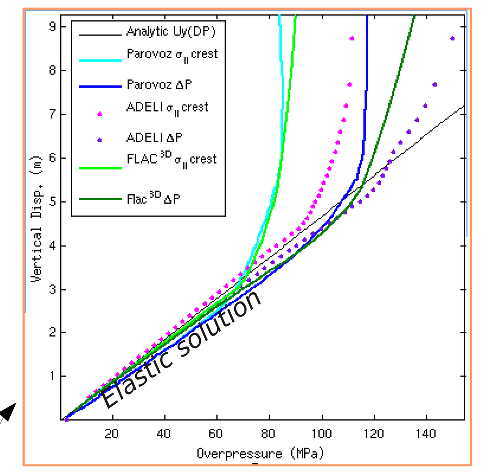
BENCHMARKS FOR EACH THREE FAILURE STAGES



Evolution of failure zones with increasing internal pressure, assuming ϕ=30°, T=5MPa, and zero dilatancy.
Tensile failure* is restricted to the first km. Wall shear failure initiates consistent with analytical prediction (ΔP ~70MPa).

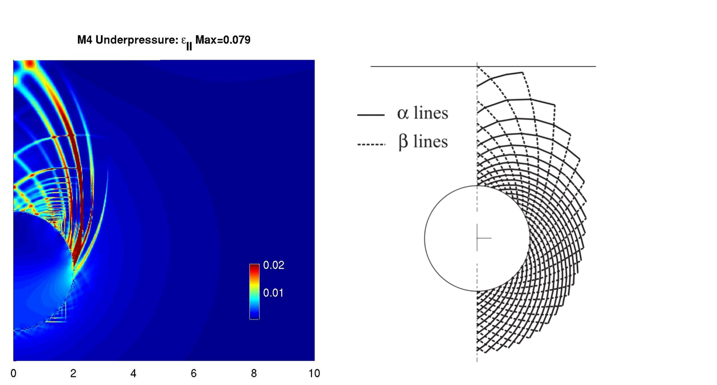
Benchmarks are made for three stages of deformation:
Stage 1: ΔP<70MPa, failure only develops from the top surface, and most of the domain remains elastic.
Stage 2: ΔP<120MPa, failure starts to propagate from the chamber wall in eccentric curves.
Stage 3: connection between both sets of faulted domains.

The complex geometry adopted by the shear bands results from rotating principal stresses from the chamber to the surface (c.f. engineering solutions). Significant departure from elastic surface displacements occurs from **Stage 3**.



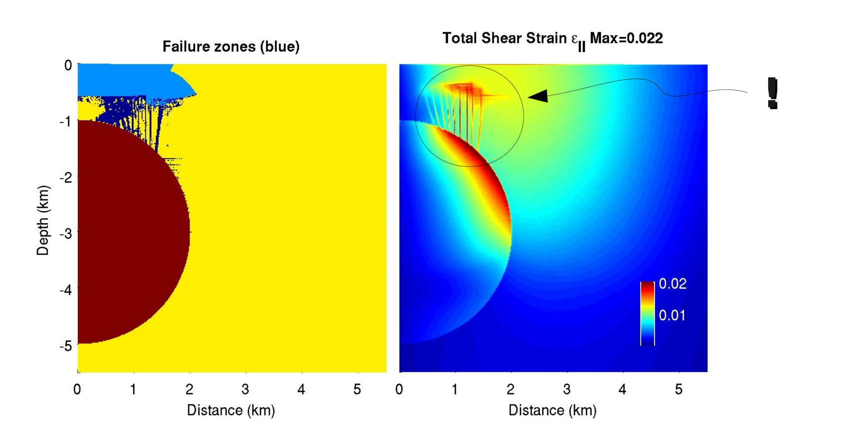
5 – MISCELLANEOUS ...

FAILURE PATTERNS FOR AN UNDERPRESSURE



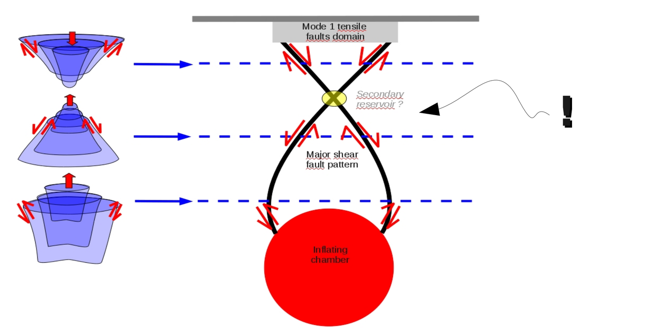
Applied underpressure ΔP=120 MPa, and friction is 20°, comparison of total shear strain with slip-lines graphical solution from *d'Escatha & Mandel, 1974*.

OVERPRESSURE IN A CHAMBER WITH H=3 KM, R=2KM



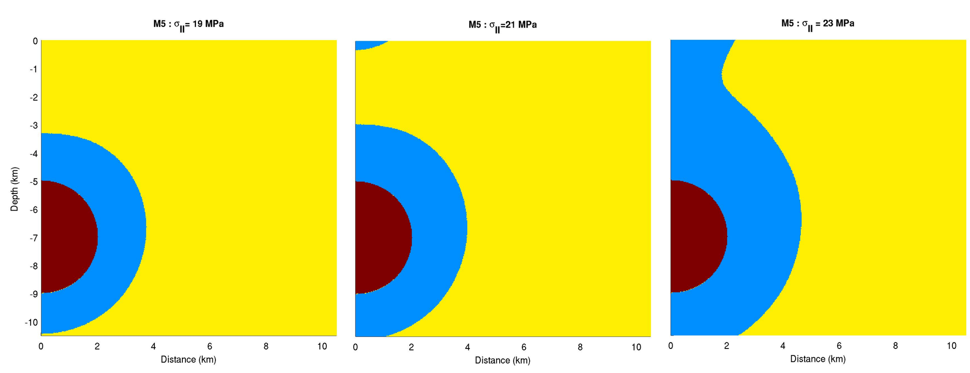
Note how sub-vertical shear sones merge with faulted tensile domain to accumulate deformation in horizontal "sills"...at 500 m depth (depends on the value of T). Thus this is not due to heterogeneity!

SCHEMATIC VIEW OF HOW FAILURE PATTERNS MAY LOOK LIKE ON A FOSSILE IGNEOUS BODY



Imbricated cone sheets depend on section depth. Note also the intermediate zone of horizontal dilation ~2km.

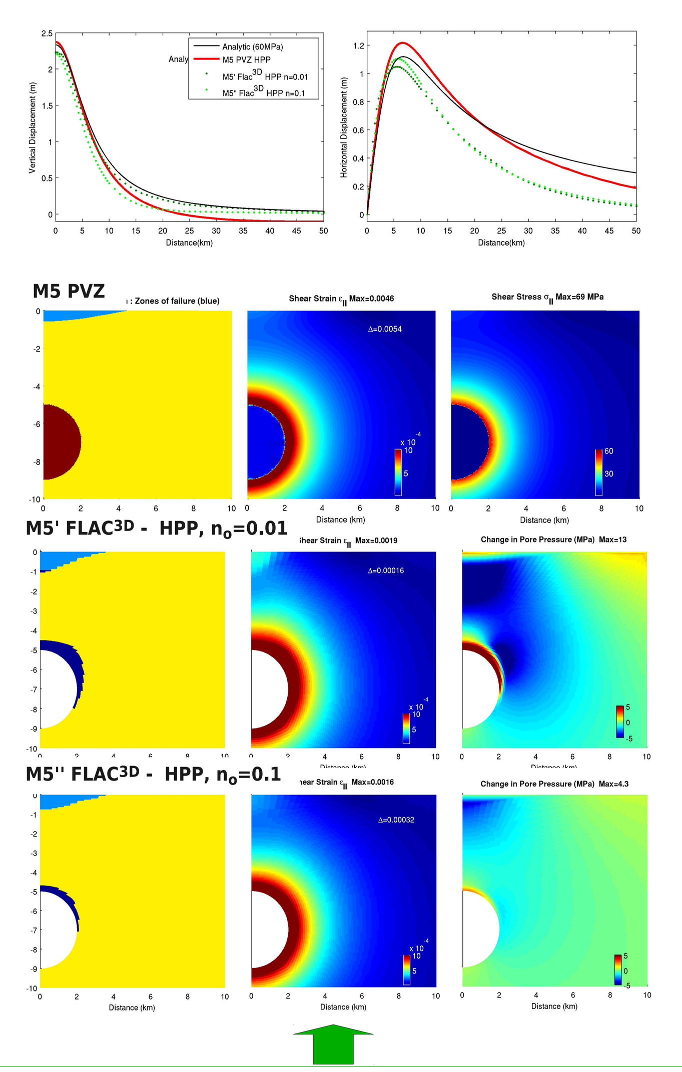
CASE WITH LITHOSTATIC BEDROCK FLUID PRESSURE & CUTOFF T= 5MPA: TENSILE FAILURE OCCURS < 20 MPa



* *In all figures displaying failure zones, dark blue zones show shear failure, and light blue zones show tensile failure.*

6- ONE & TWO WAY HYDROMECHANICAL COUPLING

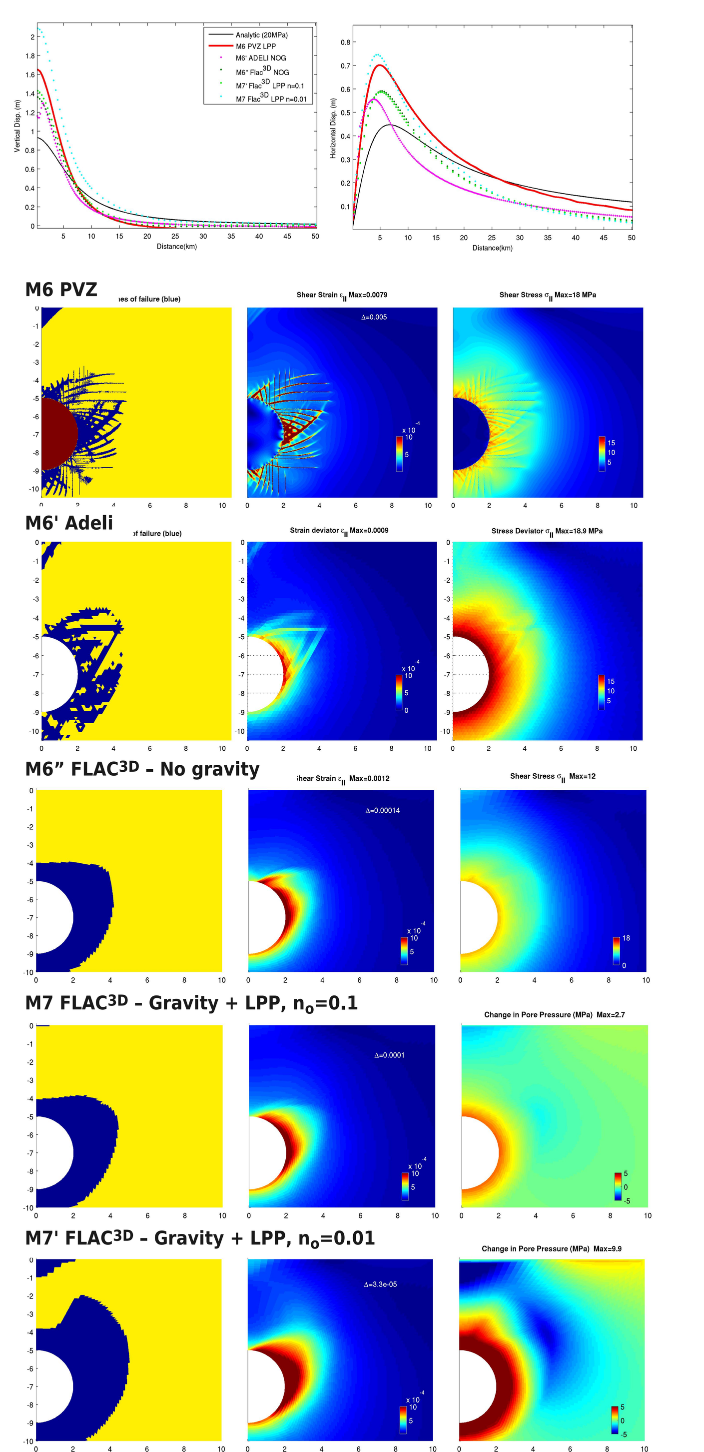
HYDROSTATIC BEDROCK FLUID PRESSURE



Models **M5** explores the effect of a hydrostatic pore fluid pressure (HPP) in the bedrock on failure propagation. T= 5 MPa, and ΔP = 60 MPa. M5 with Parovoz assumes one way coupling, whereas M5′ and M5″ with Flac3D assume two-way coupling and initial porosity n_o=0.01, 0.1.

Shear failure occurs around the chamber wall over a greater extent in M5' than in M5, related to a greater volumetric deformation ($\Delta V/V$) when porosity is low.

LITHOSTATIC BEDROCK FLUID PRESSURE



Models **M6** assume a state of lithostatic pore fluid pressure in the bedrock. Tensile failrue actually occurs if T=5MPa (see panel 5), but for benchmark here T=17.3 MPa and ΔP = 20 MPa.

Large differences of about 30% are observed between the Parovoz and the Adeli models M6 and M6′, due to the presence of an elastic chamber in the former case, and a more "rigid" yield criterion in the latter case. The Flac3D model (M6″) produces intermediate values.

A fluid-saturated bedrock of low porosity has a factor 3 increase in fluid pressure (Δp_f) enhancing propagation of the plastic domain. In contrast, a dry and porous bedrock doesn't fail.

Displacements are also sensitive to initial rock porosity (about 30% again) between the high and low porosity models. Surface deformation decreases when porosity increases, due to the absorption of "stress-induced strain" in elastic pores.

7 - CONCLUSIVE SUMMARY

1) "Mogi's" fit to geodetic data often provide too high internal overpressures...However, the onset of failure may actually occur at high overpressure, if the bedrock has few fluids in its pore spaces (and if one accounts for the gravity body force !) .

2) 3 stages of fault development, validity of elastic displacements at the surface only until Stage 3:
- tensile and normal faulting at the surface,
- shear faulting at the walls expanding eccentrically,
- connection of plastizised domain from chamber to surface. At the onset of failure, ground surface deformation may be misinterpreted by 30% if plasticity and hydromechanics are not accounted for.

3) Detail shear band geometries are obtained thanks to exceptionnally high numerical mesh resolution, and compare with tunnelling engineering results. Welcome further benchmarks! Note that changing orientations are naturally, not necessarily due to bedrock heterogeneity!

4) The discrepancies between numerical results are due to variable meshes (resolution and chamber meshing), variable yield formulation, well known sensitivity to plastic behaviour (cf. benchmarks by Buiter et al., 2008, Kaus et al., 2009).

5) Hydromechanical models indicate that rock porosity around the chamber is a key parameter that influences the change in fluid pressure, and thus the propagation of failure. A rock with low porosity is more prone to fail, with changes in fluid pressure producing greater deformation than simpler mechanical models. Cf. *Gressier et al. (Tectonophysics, 2010)* for nice experiments showing the effect of bedrock fluid pressure on intrusions geometries.

6) Our models neglect the magmatic fluid, thermal effects, heterogeneity, etc...The propagation of failure is expected to evolve rapidly into mode I as fluid filled faults will rapidly dilate. Cf. Nice paper by *White et al. (EPSL, 2011)* for seismic record of mode II dike propagation in Island.

REFERENCES

Gerbault M., Pressure conditions for shear and tensile failure around a circular magma chamber; insight from elasto-plastic modelling, **Geological Society, London, Spec. Pub., vol. 367**, pp. 111–130, In: Healy, D., Butler, R. W. H., Shipton, Z. K.& Sibson, R. H. (eds) *Faulting, Fracturing and Igneous Intrusion in the Earth's Crust*, 2012.

Gerbault M., Cappa F., Hassani R., Elasto-plastic and hydromechanical models of failure around an infinitely long magma chamber, **Geochemistry Geophysics Geosystems, vol. 13**, 2012.