Geophysical Research Abstracts
Vol. 15, EGU2013-10213-1, 2013
EGU General Assembly 2013

Complete classification of discrete resonant Rossby/drift wave triads on periodic domains

Miguel Bustamante and Umar Hayat
School of Mathematical Sciences, University College Dublin, Belfield, Dublin 4, Ireland (miguel.bustamante@ucd.ie)

We consider the set of Diophantine equations that arise in the context of the partial differential equation called "barotropic vorticity equation" on periodic domains, when nonlinear wave interactions are studied to leading order in the amplitudes. The solutions to this set of Diophantine equations are of interest in atmosphere (Rossby waves) and Tokamak plasmas (drift waves), because they provide the values of the spectral wavevectors that interact resonantly via three-wave interactions. These wavenumbers come in "triads", i.e., groups of three wavevectors.

We provide the full solution to the Diophantine equations in the physically sensible limit when the Rossby deformation radius is infinite. The method is completely new, and relies on mapping the unknown variables via rational transformations, first to rational points on elliptic curves and surfaces, and from there to rational points on quadratic forms of "Minkowski" type (such as the familiar *space-time* in special relativity). Classical methods invented centuries ago by Fermat, Euler, Lagrange, Minkowski, are used to classify all solutions to our original Diophantine equations, thus providing a computational method to generate numerically all the resonant triads in the system. Computationally speaking, our method has a clear advantage over brute-force numerical search: on a 10000^2 grid, the brute-force search would take 15 years using optimised C^{++} codes on a cluster, whereas our method takes about 40 minutes using a laptop.

Moreover, the method is extended to generate so-called quasi-resonant triads, which are defined by relaxing the resonant condition on the frequencies, allowing for a small mismatch. Quasi-resonant triads' distribution in wavevector space is robust with respect to physical perturbations, unlike resonant triads' distribution. Therefore, the extended method is really valuable in practical terms. We show that the set of quasi-resonant triads form an intricate network of connected triads, forming clusters whose structure depends on the value of the allowed mismatch. It is believed that understanding this network is absolutely relevant to understanding turbulence. We provide some quantitative comparison between the clusters' structure and the onset of fully nonlinear turbulent regime in the barotropic vorticity equation, and we provide perspectives for new research.