



A proof of concept: Airborne LIDAR–measured ellipsoidal heights of a lake surface correspond to a local geoid model

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The geoid is the theoretical model of the Earth, defined as an equipotential surface. Typically it corresponds to a mean ocean surface and is extended through the continents. Elevations are measured above “sea level” based on the fact that the surface of water in equilibrium closely follows this equipotential surface. On dry land, the geoid can be determined from gravimetric measurements, and interpolation methods are used to represent variations of gravity in a regular grid model. For practical reasons, these are represented as geoid undulation, which is the difference of the ellipsoidal height and the height of the equipotential surface.

In his work *Principia*, Isaac Newton proposed the thought experiment of connecting the North Pole and the Equator through a “canal” filled with water in order to determine gravitational flattening of the Earth. It was also Newton’s idea to use the level of water in a global network of canals and wells to survey the geoid. Of course, these experiments are impossible at a global scale, but a sufficiently large lake and an accurate method for measuring elevation independently from the geoid can be used to prove the concept.

Our objective was to measure the ellipsoidal water surface elevation of Lake Balaton in Hungary with high spatial resolution and accuracy and compare these measurements with the gravimetrically determined local geoid model. Airborne laser scanning (ALS, also known as airborne LIDAR) is a remote sensing technique capable of delivering a large number of points with elevations and horizontal positions accurate to a few centimetres. Laser pulses are emitted in a scan pattern from an airborne sensor, and are reflected from the illuminated terrain (or water) surface. Based on the position and orientation of the aircraft (typically observed by GNSS and an inertial navigation system), the scan angle and the travel time of the laser pulse, the exact position of each measurement point is calculated. In this particular case, the instrument was set to approximately collect one point for each square meter of the surveyed surface.

Lake Balaton is located in Western Hungary, with a total water surface of 597 km², and an approximate maximum length of 70 km. The main aim of the airborne survey was mapping of shore vegetation, therefore the flight plan was a series of strips following the 200 km shoreline. The relative position of the flight strip point clouds with respect to each other was adjusted by minimizing the elevation differences in overlapping areas. In order to correct horizontal positioning errors, inclined and planar reference surfaces (mostly house roofs) were used for this purpose. The resulting mean relative elevation error (standard deviation) within the full dataset is 5 mm, while the absolute elevation error is 10 cm according to specifications. Using a vector outline of the shore and a vegetation mask, data points from non-water surfaces were excluded and only the water surface elevation investigated, in approximately 80 000 000 points.

As a reference, the HGEO 2000 geoid model was used. This dataset has a planar resolution of 2 km and an accuracy of 3 ppm, and was created by an interpolation of gravimeter and variometer measurements over the territory of Hungary.

The ellipsoidal heights of the water surface of the lake showed a relief of 80 cm with a well-defined topographical pattern. Errors following the scan pattern and random errors were at least an order of magnitude smaller. The ellipsoidal height of the lake surface is lowest in the Eastern corner of the lake, showed a gradient nearly perpendicular to the axis of the lake and was the highest in the Northwest. The sigma mad (median of absolute deviation to the median) value of the water surface elevations was 20 cm. Upon correction with the geoid undulation values in the cells of the HGEO 2000 model, the modified elevations had a sigma mad of 6 cm. This proves that this large water surface in equilibrium is a very accurate representation of the local geoid.

In case of a natural lake, the question whether the water surface was in equilibrium has to be discussed. The lake lies perpendicular to the prevailing wind direction, so strong winds are known to displace the water surface and

create differences in water levels known as seiche. However, during periods of relatively calm weather, these seiche effects remain low in amplitude. The airborne survey was carried out in a period of very calm weather and over several days, so this effect is believed to be negligible. The lake has a single outflow and a main tributary supplying more than half of the incoming water, but due to strong evaporation and relatively low input, the average turnover time for water in Lake Balaton is four years. This means that no significant deviation from the gravity-controlled equilibrium is expected to result from the flow of water. The errors remaining in the dataset are explained by the elevations of artificial structures such as boats and platforms that were not masked out as non water points, by the height of waves (up to 30 cm between neighbouring ALS points in some rare cases), and the slight systematic errors encountered around the edges of the scanned strips.

Taking these errors into account, the good fit of the water surface elevation to the local geoid model has provided a proof of the concept of Newton. While this is evident based on the laws of hydrostatics, to our best knowledge, it has not yet been demonstrated by statistically sound measurements of a large natural inland water surface.