Geophysical Research Abstracts
Vol. 15, EGU2013-10381, 2013
EGU General Assembly 2013
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A high order discontinuous Galerkin method for elastic wave propagation in arbitrary heterogeneous media

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High-order numerical methods allow accurate simulations of ground motion using unstructured and relatively coarse meshes. In realistic media (sedimentary basins for example), we have to include strong variations of the material properties. For such configurations, the hypothesis that material properties are set constant in each element of the mesh can be a severe limitation since we need to use very fine meshes resulting in very small time steps for explicit time integration schemes. Moreover, smooth models are approximated by piecewise constant materials. For these reasons, we present an improvement of a nodal discontinuous Galerkin method (DG) allowing non constant material properties in the elements of the mesh for a better approximation of arbitrary heterogeneous media.

We consider an isotropic, linearly elastic two-dimensional medium (characterized by ρ, λ and μ) and solve the first-order velocity-stress system. As the stress tensor is symmetrical, let $\vec{W} = (\vec{V}, \vec{\sigma})^t$ contain the velocity vector $\vec{V} = (v_x, v_y)^t$ and the stress components $\vec{\sigma} = (\sigma_{xx}, \sigma_{yy}, \sigma_{xy})^t$, then, the system writes

$$\partial_t \vec{W} + A_x\,(\rho, \lambda, \mu)\;\partial_x \vec{W} + A_y\,(\rho, \lambda, \mu)\;\partial_y \vec{W} = 0\,,$$

where A_x and A_y are 5x5 matrices depending of the material properties.

We apply a discontinuous Galerkin method based on centered fluxes and a leap-frog time scheme to this system. We consider a bounded polyhedral domain discretized by triangles. The approximation of $\vec{W}$ is defined locally on each element by considering the Lagrange nodal interpolants.

The system is multiplied by a test function φ^t and integrated on each element T_i. To avoid computing extra terms, related to the variable properties within T_i, we introduce a change of variables on the stress components

$$\vec{\sigma} = (\sigma_{xx}, \sigma_{yy}, \sigma_{xy})^t \rightarrow \vec{\tilde{\sigma}} = \left(\frac{1}{2}(\sigma_{xx} + \sigma_{yy}), \frac{1}{2}(\sigma_{xx} - \sigma_{yy}), \sigma_{xy}\right)^t$$

which allows writing the system in a pseudo-conservative form in the variable $\vec{\tilde{W}} = (\vec{V}, \vec{\tilde{\sigma}})^t$

$$\Lambda\,(\rho, \lambda, \mu)\;\partial_t \vec{\tilde{W}} + \tilde{A}_x\;\partial_x \vec{\tilde{W}} + \tilde{A}_y\;\partial_y \vec{\tilde{W}} = 0\,,$$

where the constant matrices $\tilde{A}_x$ and $\tilde{A}_y$ do not depend anymore on the material properties and Λ is a diagonal matrix $\Lambda\,(\rho, \lambda, \mu) = diag\left(\rho, \rho, \frac{1}{\lambda+\mu}, \frac{1}{\mu}, \frac{1}{\mu}\right)$. Then, the introduction of non constant material properties inside a triangle T_i is simply realised by the calculation, via quadrature formulae, of a modified local mass matrix depending on the material properties and approximating the integral on T_i

$$\int_{T_i} \varphi_i^t\, \Lambda(\rho, \lambda, \mu)\, \partial_t \vec{W}\, dV\,.$$

The method is applied to several numerical examples including smooth velocity variations and a strong jump of the material properties and results in a clear improvement in accuracy and CPU time when compared to the initial DG method.