



A reduced order model for the solution of groundwater flow driven by randomly heterogeneous hydraulic conductivity

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The numerical solution of stochastic groundwater flow problems driven by randomly heterogeneous hydraulic conductivity distributions typically requires performing a set of computationally expensive Monte Carlo (MC) simulations. In this context, reduced order methodologies can be considered as a promising way to obtain accurate and efficient solutions of the problem. These methods are mathematical tools that enable a substantial reduction of the dimensionality of the system of algebraic equations arising from the numerical approximation of the governing Partial Differential Equations. An approximated solution is obtained via a Galerkin projection of the model equations in the space generated by a number of ad-hoc selected basis functions. The number and spatial distribution of the basis functions are the key elements driving the computational efficiency of the methodology and controlling the accuracy of the solution. Computation of the basis functions is the most time-expensive step of the procedure because it requires the solution of the complete set of model equations for some realizations of the random parameter distributions. Given a set of MC spatial realizations of the random hydraulic conductivity field, a greedy algorithm is employed to select a number of realizations upon which we appropriately build the reduced order model. Starting from one basis function, the greedy algorithm computes the reduced-model solution for all of the selected MC realizations. The full system model solution is then computed for the realization associated with the largest error between the reduced- and full-model solutions. A new basis function is then obtained by orthonormalization of the full system model solution with respect the first basis function employed. The set of basis functions is enriched upon repeating the procedure until the largest error is lower than a predefined tolerance. We apply this technique to solve a two-dimensional steady-state saturated groundwater flow scenario in the presence of a uniform (in the mean) hydraulic head gradient. Hydraulic conductivity is modeled as a second-order stationary spatially correlated random field. We explore the sensitivity of the greedy algorithm to the parameters controlling the distribution of hydraulic conductivity and to the error tolerance in terms of (a) the number of basis functions, and (b) the differences between full model and reduced model MC empirical distributions. Our results show that the reduced model procedure is accurate and efficient when the conductivity field is characterized by a small variance and/or a large integral scale. When the system is associated with large variance and small integral scale, the number of basis functions required to accurately describe the ensemble of MC solutions increases dramatically, thus reducing significantly the computational advantages associated with the reduced model.