Geophysical Research Abstracts
Vol. 15, EGU2013-11473, 2013
EGU General Assembly 2013

A study of the numerical limitations of a homogenous rectangular prism formula

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In computing accurately the total gravitational attraction of arbitrarily complicated source body, the widely used standard method is to approximate it by a sum of attractions from (possibly many) disjoint rectangular prisms of constant density. Such methods are notorious for being computationally slow but regarded as being arbitrarily accurate; the finer the prisms that describe the source body the better the approximation of the total gravitational attraction.

Mathematically, the attraction of the individual prisms is computed using the so-called rectangular homogenous prism formula which is a definite volume integral (i.e. a triple integration). The standard application (in Cartesian coordinates) involves an alignment of the Cartesian frames of reference of the field point and the source point support, so that the x-, y- and z-axis of the aligned Cartesian reference frames are also aligned with the sides of the prism. In computing the prism attraction the values of a primitive function are determined for the coordinates of the prism corners. In short, the definite integral involves the alternating subtraction and addition of the values of a primitive function. On a computer, the values of a primitive function are truncated by a finite representation (32BIT, 64BIT). This truncated representation of a function results in numerical instabilities of the rectangular prism formula; i.e. the value of a primitive function for each corner point of the prism is accurate, but not the final definite integral.

In this study we will quantify this numerical instability of the homogenous prism formula for the gravitational potential, for the components of the gravitational vector, and for all the components of the 2nd order tensor of the gravitational gradients. Furthermore, we will investigate how the quadruple precision (128 BIT representation of reals on a computer) improves the stability of the prism formula.

In practical terms, the numerical instability of the prism formula puts limitations on how well the sum of the gravitational attractions of a large number of fine homogenous rectangular prisms describing a complicated source will approximate the total gravitational attraction of a complicated source body.