



Nonlinear resonances and mixing in a simple shallow lake model

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Large-scale transport in environmental flows is often dominantly determined by the velocity field of the flow. Diffusion of certain quantities, like pollutants and temperature, can be neglected with respect to advective transport. Understanding the topological features of the velocity field is thus very important for the qualitative analysis of the large-scale mixing properties of these passive scalars in water bodies. Large horizontal circulating zones (often called gyres) are prevalent structures of wind induced shallow lake flows. In this presentation we analyse the currents generated by wind in a square shaped shallow lake. In case of a steady flow field, induced by a time-independent wind stress field, the typical flow pattern consists of two counter-rotating gyres. When applying periodic disturbances in the wind stress field, mixing regions of different widths develop between the gyres. This region is filled with coherent structures, strongly increasing advective transport in the lake. Meanwhile, the inner regions of the gyres remain stable; their outer periodic orbits serve as transport barriers. Our statement is that the width of the mixing region reaches its maximum at a certain scale of wind disturbance frequencies. This characteristic frequency scale corresponds to the typical circulation frequencies of the gyres. Our flow model consists of a two dimensional, depth-averaged flow field of the volume preserving water body with wind surface stress. The flow has a stream function that satisfies the linearised shallow-water vorticity transport equation. This corresponds to a Hamiltonian system, where the stream function plays the role of the Hamiltonian. In the steady state the gyres consist of periodic orbits, so this is an (one degree of freedom) integrable mechanical system, like the undamped pendulum. In the periodically disturbed case the system remains Hamiltonian with a topological similarity to the phase portrait of the forced pendulum. Thus we can analyse the hydraulic transport properties of the lake by the Hamiltonian formalism of dynamical systems theory, where the volume of the flow is the conserved quantity, and the velocity field is a Hamiltonian vector field. Larger mixing zones appear via the overlapping of nonlinear resonant bands, whenever the circulation and the wind disturbance frequencies are in some (e.g. 1:1) rational relation.