Instability of a shear flow around a circular island and emission of vortex dipoles

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Laboratory experiments suggest that the instability of a circular flow with alternating vorticity around a cylinder culminates in shedding of two or three vortex dipoles. These observations lead us to the following questions: What are the conditions for a flow around a cylinder (island) to be stable? If the flow is unstable, how long can it survive, i.e. what are the growth rates of its unstable modes, and what vortical patterns emerge due to the instability? This topic is of oceanographic significance, since coastal waters near islands are usually rich in chemical and biological material, whereas vortex dipoles are normally quite robust and can serve as carriers of the trapped material.

To answer the above-posted questions we apply both analytical and numerical approaches. First we establish analytically the conditions for the linear instability of a two-dimensional azimuthally symmetric flow around a rigid circular boundary. To simplify the problem, we assume the unperturbed velocity profile to correspond to two concentric neighboring rings of uniform but different vorticity, with the inner ring touching the cylinder. As the next step, based on the analytical results, we carry out numerical simulations aiming to study the long-term, nonlinear evolution of the linearly unstable flow and to follow the formation of new vortical patterns.

At the analytical stage, we remain in the contour-dynamics framework, i.e. regard the fluid to be inviscid and consider the perturbations that affect the free edges of the two rings only. At the numerical stage, we run contour-dynamics simulations, which are made feasible by using the Green function for the Laplace operator on a plane with a circular cut-out, and high Reynolds number simulations. In the latter case, a finite-element Navier–Stokes module of COMSOL software is used.

Fixing the ratio of the vorticity in the two rings, assuming the scaled cylinder radius to be 1, and denoting the outer radii of the inner and outer vorticity rings a and b, we determine analytically the regions on the (a, b)-plane, where azimuthal modes m = 2, 3, . . . are unstable, and calculate the growth rates of these modes as functions of a and b. For each mode m > 2, apart from the region where this mode and all the lower modes are unstable (we call it the regular instability region), a closed finite-size region is found, in which only mode m is unstable (the unique instability region). In the latter case, the inner ring is much thinner than the outer one. For the ordinary instability regions, the two numerical techniques yield qualitatively the same result, the emergence of m dipoles. However, for the unique instability regions, no dipole emergence is observed in the contour-dynamics simulations.

This approach is extended to the two-layer quasigeostrophic model, where one vorticity ring is located in the upper layer, and the other ring, in the lower layer. Here, as in the barotropic case, both analytical and numerical methods are exploited.