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Numerical Modeling of Deep Mantle Convection: Advection and Diffusion Schemes for Marker Methods

Elvira Mulyukova (1), Marcin Dabrowski (2,3), Bernhard Steinberger (1,3)

(1) GFZ German Research Center for Geosciences, Physics of the Earth, Potsdam, Germany (elvira5678@gmail.com), (2) Computational Geology Laboratory, Polish Geological Institute - National Research Institute, Wroclaw, Poland, (3) Physics of Geological Processes, University of Oslo, Oslo, Norway

Thermal and chemical evolution of Earth's deep mantle can be studied by modeling vigorous convection in a chemically heterogeneous fluid. Numerical modeling of such a system poses several computational challenges. Dominance of heat advection over the diffusive heat transport, and a negligible amount of chemical diffusion results in sharp gradients of thermal and chemical fields. The exponential dependence of the viscosity of mantle materials on temperature also leads to high gradients of the velocity field. The accuracy of many numerical advection schemes degrades quickly with increasing gradient of the solution, while the computational effort, in terms of the scheme complexity and required resolution, grows. Additional numerical challenges arise due to a large range of length-scales characteristic of a thermochemical convection system with highly variable viscosity. To examplify, the thickness of the stem of a rising thermal plume may be a few percent of the mantle thickness. An even thinner filament of an anomalous material that is entrained by that plume may consitute less than a tenth of a percent of the mantle thickness.

We have developed a two-dimensional FEM code to model thermochemical convection in a hollow cylinder domain, with a depth- and temperature-dependent viscosity representative of the mantle (Steinberger and Calderwood, 2006). We use marker-in-cell method for advection of chemical and thermal fields. The main advantage of perfoming advection using markers is absence of numerical diffusion during the advection step, as opposed to the more diffusive field-methods. However, in the common implementation of the marker-methods, the solution of the momentum and energy equations takes place on a computational grid, and nodes do not generally coincide with the positions of the markers. Transferring velocity-, temperature-, and chemistry- information between nodes and markers introduces errors inherent to inter- and extrapolation.

In the numerical scheme that we use for this study, the velocity field is discretised using second order triangular elements, which gives second order accuracy of interpolation from grid-nodes to markers. A fourth order Runge-Kutta solver is used to compute marker-trajectories. We reevaluate the velocity field for each of the intermediate steps of the ODE-solver, rendering our advection scheme to be fourth-order accurate in time.

We compare two different approaches for performing the thermal diffusion step. In the first, more conventional approach, the energy equation is solved on a static grid. For this grid, we use first-order triangular elements and a higher resolution than for the velocity-grid, to compensate for the lower order elements. The temperature field is transferred between grid-nodes and markers, and a subgrid diffusion correction step (Gerya and Yuen, 2003) is included to account for the different spatial resolutions of the markers and the grid. In the second approach, the energy equation is solved directly on markers. To do this, we compute a constrained Delaunay triangulation, with markers as nodes, at every time step.

We wish to resolve the large range of spatial scales of the solution at lowest possible computational cost. In several existing codes this is achieved with dynamically adaptive meshes, which use high resolution in regions with high solution gradients, and vice versa. The numerical scheme used in this study can be extended to include a similar feature, by regenerating the thermal and mechanical grids in the course of computation, adapting them to the temperature and chemistry fields carried by the markers.

We present the results of thermochemical convection simulations obtained using the schemes outlined above, as well as the results of the numerical benchmarks commonly used in the geodynamics community. The quality of the solutions, as well as the computational cost of our schemes, are discussed.