



Quantification of Long-Range Persistence in Geophysical Time Series

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Time series in the Earth Sciences are often characterized as self-affine long-range persistent, where the power spectral density, S , exhibits a power law dependence on frequency, f , $S(f) \sim f^{-\beta}$, with β the persistence strength. For modelling purposes, it is important to determine the strength of self-affine long-range persistence β as precisely as possible and to quantify the uncertainty of this estimate. Here we compare four common analysis techniques for quantifying self-affine long-range persistence: (a) rescaled range (R/S) analysis, (b) semivariogram analysis, (c) detrended fluctuation analysis, and (d) power spectral analysis. To evaluate these methods, we construct ensembles of synthetic self-affine noises and motions with different (i) time series lengths $N = 64, 128, 256, \dots, 131,072$, (ii) modelled persistence strengths $\beta_{\text{model}} = -1.0, -0.8, -0.6, \dots, 4.0$, and (iii) one-point probability distributions (Gaussian, log-normal: coefficient of variation $c_v = 0.2 - 2.0$, Levy: tail parameter $a = 1.0 - 1.9$) and evaluate the four techniques by statistically comparing their performance. Over 17,000 sets of parameters are produced, each characterising a given process; for each process type, 100 realisations are created. The four techniques give the following results in terms of systematic error (bias = average performance test results for β over 100 realisations minus modelled β) and random error (standard deviation of measured β over 100 realisations): (i) Hurst rescaled range (R/S) analysis is not recommended to use due to large systematic errors. (ii) Semivariogram analysis shows no systematic errors but large random errors for self-affine noises with $1.2 \leq \beta \leq 2.8$. (iii) Detrended fluctuation analysis is well-suited for time series with thin-tailed probability distributions and for persistence strengths of $\beta \geq 0.0$. (iv) Spectral techniques perform the best of all four techniques: for self-affine noises with positive persistence ($\beta \geq 0$) and symmetric one-point distributions they have no systematic errors and compared to the other three techniques, small random errors; for anti-persistent self-affine noises ($\beta < 0$) and asymmetric one-point probability distributions, spectral techniques have small systematic and random errors. We finish this paper by discussing long-range persistence of three geophysical time series—palaeotemperature, river discharge, and Auroral electrojet index—with the three representing three different types of probability distribution—Gaussian, log-normal, and Levy, respectively.