



The Curvelet Transform in the analysis of 2-D GPR data: Signal enhancement and extraction of orientation-and-scale-dependent information

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The Ground Probing Radar (GPR) has become a valuable means of exploring thin and shallow structures for geological, geotechnical, engineering, environmental, archaeological and other work. GPR images usually contain geometric (orientation/dip-dependent) information from point scatterers (e.g. diffraction hyperbolae), dipping reflectors (geological bedding, structural interfaces, cracks, fractures and joints) and other conceivable structural configurations. In geological, geotechnical and engineering applications, one of the most significant objectives is the detection of fractures, inclined interfaces and empty or filled cavities frequently associated with jointing/faulting. These types of target, especially fractures, are usually not good reflectors and are spatially localized. Their scale is therefore a factor significantly affecting their detectability. At the same time, the GPR method is notoriously susceptible to noise. Quite frequently, extraneous (natural or anthropogenic) interference and systemic noise swamp the data with unusable information that obscures, or even conceals the reflections from such targets. In many cases, the noise has definite directional characteristics (e.g. clutter). Raw GPR data require post-acquisition processing, as they usually provide only approximate target shapes and distances (depths).

The purpose of this paper is to investigate the Curvelet Transform (CT) as a means of S/N enhancement and information retrieval from 2-D GPR sections (B-scans), with particular emphasis placed on the problem of recovering features associated with specific temporal or spatial scales and geometry (orientation/dip).

The CT is a multiscale and multidirectional expansion that formulates a sparse representation of the input data set (Candès and Donoho, 2003a, 2003b, 2004; Candès et al., 2006). A signal representation is sparse when it describes the signal as a superposition of a small number of components. What makes the CT appropriate for processing GPR data is its capability to describe wavefronts. The roots of the CT are traced to the field of Harmonic Analysis, where curvelets were introduced as expansions for asymptotic solutions of wave equations (Smith, 1998; Candès, 1999). In consequence, curvelets can be viewed as primitive and prototype waveforms – they are local in both space and spatial frequency and correspond to a partitioning of the 2D Fourier plane by highly anisotropic elements (for the high frequencies) that obey the parabolic scaling principle, that their width is proportional to the square of their length (Smith, 1998).

The GPR data essentially comprise recordings of the amplitudes of transient waves generated and recorded by source and receiver antennae, with each source/receiver pair generating a data trace that is a function of time. An ensemble of traces collected sequentially along a scan line, i.e. a GPR section or B-scan, provides a spatio-temporal sampling of the wavefield which contains different arrivals that correspond to different interactions with wave scatterers (inhomogeneities) in the subsurface. All these arrivals represent wavefronts that are relatively smooth in their longitudinal direction and oscillatory in their transverse direction. The connection between Harmonic Analysis and curvelets has resulted in important nonlinear approximations of functions with intermittent regularity (Candès and Donoho, 2004). Such functions are assumed to be piecewise smooth with singularities, i.e. regions where the derivative diverges. In the subsurface, these singularities correspond to geological inhomogeneities, at the boundaries of which waves reflect. In GPR data, these singularities correspond to wavefronts. Owing to their anisotropic shape, curvelets are well adapted to detect wavefronts at different angles and scales because aligned curvelets of a given scale, locally correlate with wavefronts of the same scale.

The CT can also be viewed as a higher dimensional extension of the wavelet transform: whereas discrete wavelets are designed to provide sparse representations of functions with point singularities, curvelets are designed to provide sparse representations of functions with singularities on curves.

This work investigates the utility of the CT in processing noisy GPR data from geotechnical and archaeo-

metric surveys. The analysis has been performed with the Fast Discrete CT (FDCT – Candès et al., 2006) available from <http://www.curvelet.org/> and adapted for use by the matGPR software (Tzanis, 2010). The adaptation comprises a set of driver functions that compute and display the curvelet decomposition of the input GPR section and then allow for the interactive exclusion/inclusion of data (wavefront) components at different scales and angles by cancelation/restoration of the corresponding curvelet coefficients. In this way it is possible to selectively reconstruct the data so as to abstract/retain information of given scales and orientations.

It is demonstrated that the CT can be used to: (a) Enhance the S/N ratio by cancelling directional noise wavefronts of any angle of emergence, with particular reference to clutter. (b) Extract geometric information for further scrutiny, e.g. distinguish signals from small and large aperture fractures, faults, bedding etc. (c) Investigate the characteristics of signal propagation (hence material properties), albeit indirectly. This is possible because signal attenuation and temporal localization are closely associated, so that scale and spatio-temporal localization are also closely related. Thus, interfaces embedded in low attenuation domains will tend to produce sharp reflections rich in high frequencies and fine-scale localization. Conversely, interfaces in high attenuation domains will tend to produce dull reflections rich in low frequencies and broad localization.

At a single scale and with respect to points (a) and (b) above, the results of the CT processor are comparable to those of the tuneable directional wavelet filtering scheme proposed by Tzanis (2013). With respect to point (c), the tuneable directional filtering appears to be more suitable in isolating and extracting information at the lower frequency – broader scale range.

References

Candès, E., 1999. Harmonic analysis of neural networks. *Appl. Comput. Harmon. Anal.*, 6, 197-218.

Candès, E. and Donoho, D., 2003a. Continuous curvelet transform: I. Resolution of the wavefront set. *Appl. Comput. Harmon. Anal.*, 19, 162-197.

Candès, E. and Donoho, D., 2003b. Continuous curvelet transform: II. Discretization and frames. *Appl. Comput. Harmon. Anal.*, 19, 198-222.

Candès, E. and Donoho, D., 2004. New tight frames of curvelets and optimal representations of objects with piecewise C2 singularities. *Comm. Pure Appl. Math.*, 57, 219-266.

Candès, E. J., L. Demanet, D. L. Donoho, and L. Ying, 2006. Fast discrete curvelet transforms (FDCT). *Multiscale Modeling and Simulation*, 5, 861–899.

Smith, H. F., 1998. A Hardy space for Fourier integral operators. *Journal of Geometric Analysis*, 7, 629 – 653.

Tzanis, A., 2010. matGPR Release 2: A freeware MATLAB® package for the analysis & interpretation of common and single offset GPR data. *FastTimes*, 15 (1), 17 – 43.

Tzanis, A., 2013. Detection and extraction of orientation-and-scale-dependent information from two-dimensional GPR data with tuneable directional wavelet filters. *Journal of Applied Geophysics*, 89, 48-67. DOI: 10.1016/j.jappgeo.2012.11.007