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Modeling Groundwater Flow using both Neumann and Dirichlet Boundary Conditions

Wouter Zijl (1), Mustafa El-Rawy (1,2), Okke Batelaan (1,3,4)

(1) Dept. of Hydrology and Hydraulic Engineering, Vrije Universiteit Brussel, Brussels, Belgium, (2) Dept. of Civil Engineering, Minia University, Minia, Egypt, (3) Dept. of Earth and Environmental Sciences, KU Leuven, Heverlee, Belgium, (4) School of the Environment, Flinders University, Adelaide, Australia

In groundwater flow models it is customary to use the recharge rate, obtained from measured precipitation minus run off and evapotranspiration, as the top boundary condition (a Neumann boundary condition). However, as has been emphasized by Tóth (1962; 2009), the topography of the water table offers a better boundary condition (a Dirichlet boundary condition), because it leads to the delineation of flow systems and stagnation zones.

However, in practical modeling studies the recharge rates obtained when using the Dirichlet boundary condition may turn out to be unrealistically small or large. To remediate this we have developed an unconventional modeling procedure that is based on both the Neumann and the Dirichlet boundary condition on the phreatic surface. Such a model does not only calculate the heads and fluxes, but also an update of the initially perceived hydraulic conductivities, in such a way that the initially perceived conductivity model is preserved as much as possible.

For given grid block conductivities, numerical groundwater models (e.g. MODFLOW) are linear in the heads. However, for given heads the numerical models are not linear in the grid block conductivities. Mohammed et al. (2009) have developed a MODFLOW-compatible numerical model that is linear in the stream functions for given grid block conductivities, while it is also linear in the grid block resistivities (inverse of conductivities) if the heads are given.

Unconventional modeling is based on this bi-linearity. Assume we specify a reasonable perception of the hydraulic conductivities and determine the numerical solution with Neumann boundary conditions. The resulting fluxes are then substituted into the stream function model, together with Dirichlet boundary conditions, and the grid block resistivities can then be determined by a standard routine for solving systems of linear algebraic equations. The thus calibrated grid block conductivities do not deviate much from the initially perceived conductivity model and honor all the Dirichlet and Neumann boundary data. This so called Constrained Back Projection (CBP) has been developed by Mohammed et al. (2009) and exemplified for synthetic problems. The method is well suited to determine conductivities in ten to hundreds of zones, but solving the algebraic system for thousands to millions of grid block conductivities becomes problematic.

A related idea has already been proposed in the 1980s by Wexler (Wexler et al., 1985; Yorkey and Webster, 1987; Kohn and Vogelius, 1987; Wexler, 1988; Kohn and McKenney, 1990) in the context of electric impedance tomography for geophysical and medical imaging. El-Rawy et al. (2010, 2011) has developed and validated this so-called Double Constraint Method (DCM) in the context of hydrogeology and groundwater flow, with applications to two case studies in Belgium. DCM can handle MODFLOW models with thousands to millions of grid block conductivities, but is not very suitable for zonation and is, therefore, complementary to CBP.

Application of DCM under a number of different hydrogeological conditions makes the estimate of the hydraulic conductivities more accurate by using a Kalman Filter.