



On the reproducing kernel for an oblate ellipsoid of revolution and its use in gravity field studies: Series representation, summation and numerical treatment

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In the introductory part the importance of the topic for gravity field studies is outlined. Some concepts and tools often used for the representation of the solution of the related boundary value problems are mentioned. Subsequently a weak formulation of Neumann's problem is considered with emphasis on a particular choice of function basis generated by the reproducing kernel of the respective Hilbert space of functions. The use of the reproducing kernel offers a very straightforward way leading to entries in Galerkin's matrix of the respective linear system for unknown scalar coefficients. The paper then focuses on the construction of the reproducing kernel for the solution domain given by the exterior of an oblate ellipsoid of revolution. The fundamental problem, however, is the possibility of practical summation of the series that represents the kernel. It is difficult to reduce the number of summation indices since in the ellipsoidal case there is not a straightforward analogue to the addition theorem known for spherical situation. This, in consequence, makes the computation of the kernel and especially the set of the entries in Galerkin's matrix, even by means of high performance facilities, rather demanding. The reproducing kernel, its series representation and structure are analyzed. The apparatus of hypergeometric functions is of essential importance. Subsequently the kernel is split into parts. Some of the resulting series may be summed in a relatively straightforward way, apart from necessary technical tricks. However, there are also series that have to be treated in this connection, their summation needs a more complex tools. Partial fractions were applied first with a relatively good result but subsequently the summation was converted to elliptic integrals. This solution is discussed in details and resulted in an effective numerical treatment of the kernel.