



## Extracting Information about Model Structure from Observations

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We propose an expectation-maximization (EM) approach to adapting the state-transition component of a hidden Markov model. The state-transition simulator is emulated by sparse Gaussian process regression (sgpr; Snelson and Ghahramani 2006), and the expectation step consists of Bayesian smoothing of the emulator state conditional on observations taken according to a known observation function. After smoothing, maximum likelihood sgpr hyperparameters and pseudo-inputs are estimated. Expectation and maximization steps are iterated until convergence of the state estimates. The method results in a posterior emulator of the state-transition model rather than an input-output relationship developed by traditional neural network and regression approaches. This allows for a direct comparison between prior and posterior state-transition response surfaces, which can facilitate subjective and/or objective evaluations of model deficiencies.

The method was tested on a simple rainfall-runoff simulator by assimilating daily observations of streamflow using the ensemble Kalman smoother (Evensen and van Leeuwen, 2000). The Nash-Sutcliffe efficiency of predictions made by the emulator for the simulation period containing observations increased monotonically after each maximization step for a total increase of 0.72 to 0.98. Similarly, the efficiency of the trained emulator increased from 0.60 to 0.80 during a validation period not used for training.

We calculated the Kullback-Leibler divergence between prior and posterior response surfaces and from this identified missing or incorrect conceptual model components. Major changes to the model structure during EM training included an addition of a process which mimicked drainage, prolonged infiltration after a rainfall event, and a change in the slope of the runoff ratio function which served to increase runoff only during high precipitation events.

Snelson, E., & Ghahramani, Z. (Eds.) (2006). *Sparse Gaussian Processes using Pseudo-inputs*. Cambridge, MA: MIT Press

Evensen, G., & van Leeuwen, P.J. (2000). An ensemble Kalman smoother for nonlinear dynamics. *Monthly Weather Review*, 128, 1852-1867