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New form of the Hamiltonian equations for the nonlinear water-wave problem, based to a new representation of DtN operator, and some applications

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After the recognition by Petrov and Zakharov that the dynamics of the nonlinear water-wave problem may be treated by means of Hamilton's Principle, a lot of work has been performed in order to exploit the Hamiltonian structure for the theoretical and numerical study of the problem [1], [2]. In this approach the dynamical equations of the problem are formulated in terms of the canonical variables $\eta = \eta(\mathbf{x}, t)$ (the free surface elevation) and $\varphi = \varphi(\mathbf{x}, t)$ (the trace of the velocity potential on the free-surface). The main difficulty in this approach is the treatment of the Dirichlet-to-Neumann (DtN) operator, necessary in order to implement the internal kinematics of the liquid domain. Functional (Volterra-Taylor) series expansion of DtN operator has been introduced and implemented in the Fourier space [3], in order to make applicable this approach to practical problems. However, in the case of a general bathymetry, this approach becomes very complicated and has been implemented up to now only for small depth variations [4].
Luke introduced another variational principle for the nonlinear water-wave problem, free of the essential constraint of the internal kinematics. Based on this principle, in conjunction with an accurate representation of the velocity potential in terms of local vertical modes, Athanassoulis et al. developed a system of horizontal equations with respect to modal amplitudes $\varphi_n(\mathbf{x}, t),\ n = -2, -1, 0, ...$, [5], [6], successfully applied to a variety of problems with varying bathymetry [7]. The main novelty of this approach is the introduction of two unconventional modal amplitudes $\varphi_{-2},\ \varphi_{-1}$ ensuring rapid convergence of the modal series, and the exact satisfaction of the kinematic conditions on the free surface and the seabed, even in places where the slope of these surfaces is high.
In the present work, a careful reanalysis of the second approach has been performed, revealing that the DtN operator can be expressed solely by means of the free-surface mode φ_{-2}. This representation is very efficient numerically and easily applicable to any bathymetric configuration. As a first application of the new exact water-wave equations, we present a unified numerical treatment of nonlinear steady travelling waves, corresponding to a wide range of amplitudes and water depths.

References:

[1] W. Craig, C. Sulem, J. of Comp. Phys. 108, 73-83 (1993)
[2] W. Craig, C.E. Wayne, Russian Math. Surveys 62:3, pp. 453-473 (2007)
[3] D.P. Nicholls, J. Comp. Phys., 143, pp. 224-240, (1998)
[4] W. Craig, P. Guyenne and C. Sulem, Proceedings of the 19th International Offshore and Polar Engineering Conference Osaka, Japan, June 21-26, (2009)
[5] G.A. Athanassoulis, K.A. Belibassakis, Journal of Fluid Mechanics, 389, pp. 275-301, (1999)
[6] G.A. Athanassoulis, K.A. Belibassakis, J. Discrete Cont. Dyn. Systems, Supplement, pp. 75-84, (2007).
[7] K.A. Belibassakis, G.A. Athanassoulis, Coastal Engineering, 58, pp. 337-350, (2011)