



Adaptive random renormalization group classification of multiscale dispersive processes

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Renormalization group operators provide a detailed classification tool for dispersive processes. We begin by reviewing a two-scale renormalization group classification scheme. Repeated application of one operator is associated with long time behavior of the process while repeated application of the other is associated with short time behavior. This approach is shown to be robust even in the presence of non-stationary increments and/or infinite second moments. Fixed points of the operators can be used for further sub classification of the processes when appropriate limits exist. As an example we look at advective dispersion in an ergodic velocity field.

Let $X(t)$ be a fixed point of the long-time renormalization group operator (RGO) $RX(t)=X(rt)/r^p$. Scaling laws for the probability density, mean first passage times, and finite-size Lyapunov exponents of such fixed points are reviewed in anticipation of more general results. A generalized RGO, R_p , where the exponent in R above is now a random variable is introduced. Scaling laws associated with these random RGOs (RRGOs) are demonstrated numerically and applied to a process modeling the transition from sub-dispersion to Fickian dispersion. The scaling laws for the RRGO are not simple power laws, but instead are a weighted average of power laws. The weighting in the scaling laws can be determined adaptively via Bayes' theorem.