



## Achieving seventh-order amplitude accuracy in leapfrog integrations

Paul Williams

Department of Meteorology, University of Reading, Reading, United Kingdom (p.d.williams@reading.ac.uk)

The leapfrog time-stepping scheme makes no amplitude errors when integrating linear oscillations. Unfortunately, the Robert–Asselin filter, which is used to damp the computational mode, introduces first-order amplitude errors. The RAW filter, which was recently proposed as an improvement, eliminates the first-order amplitude errors and yields third-order amplitude accuracy. However, it has not previously been shown how to further improve the accuracy by eliminating the third- and higher-order amplitude errors.

Here, it is shown that leapfrogging over a suitably weighted blend of the filtered and unfiltered tendencies eliminates the third-order amplitude errors and yields fifth-order amplitude accuracy. It is further shown that the use of a more discriminating  $(1, -4, 6, -4, 1)$  filter instead of a  $(1, -2, 1)$  filter eliminates the fifth-order amplitude errors and yields seventh-order amplitude accuracy. The proposed new schemes are tested successfully in numerical integrations of a simple nonlinear system. They appear to be attractive alternatives to the filtered leapfrog schemes currently used in many atmosphere and ocean models.

### Reference

PD Williams (2013) Achieving seventh-order amplitude accuracy in leapfrog integrations. *Monthly Weather Review*, in press.