



Nonlinear progressive acoustic-gravity waves: Exact solutions

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We consider finite-amplitude mechanical waves in an inhomogeneous, compressible fluid in a uniform gravity field. The fluid is assumed to be inviscid, and wave motion is considered as an adiabatic thermodynamic process. The fluid either occupies an unbounded domain or has free and/or rigid boundaries. Wave motion is described by the momentum, continuity, and state equations in Lagrangian coordinates. We consider generic inhomogeneous fluids; no specific assumptions are made regarding the equation of state or spatial variations of the mass density or the sound speed in the absence of waves. The density and the sound speed are piece-wise continuous functions of position. The discontinuities represent fluid-fluid interfaces, such as the air-sea interface. Following a recent work on linear acoustic-gravity waves [O. A. Godin, Incompressible wave motion of compressible fluids, *Phys. Rev. Lett.*, 108, 194501 (2012)], here we investigate a particular class of non-linear wave motions in fluids, in which pressure remains constant in each moving fluid parcel. Exact, analytic solutions of the non-linear hydrodynamics equations are obtained for two distinct scenarios. In the first scenario, the fluid is either unbounded or has a free surface. In the latter case, the exact analytic solution can be interpreted as a progressive surface wave. In the second scenario, the fluid has a free surface and a sloping, plane rigid boundary. Then the exact analytic solution represents an edge wave propagating horizontally along the rigid boundary. In both scenarios, the flow field associated with the finite-amplitude waves is rotational. When the sound speed tends to infinity, our results reduce to well-known finite-amplitude waves in incompressible fluids. In another limit, when the wave amplitude tends to zero, the exact solutions reduce to known results for linear waves in compressible fluids. The possibility of extending the theory to rotating fluids and fluids with a shearing background flow will be examined. Implications of the theoretical results for coupling of non-linear acoustic-gravity waves in the oceans and atmosphere will be discussed.