



Solutions for the diurnally forced advection-diffusion equation to estimate

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Differences in the diurnal variations of temperature at different depths in streambed sediments are commonly used for estimating vertical fluxes of water in the streambed. We applied spatial and temporal rescaling of the advection-diffusion equation to derive two new explicit analytical relationships to calculate the fluid fluxes and the effective thermal diffusivity of the water-sediment matrix from streambed temperature measurements. The first equation provides a direct estimate of the advective component of the thermal flux and the second equation calculates the thermal diffusivity directly from amplitude decay, phase delay, and sensor depth information. In the absence of depth information, the Peclet number can be calculated from just the amplitude and phase information. Where current practice requires a priori estimation of streambed thermal diffusivity, the new approach allows an independent calculation, improving precision of estimates. Furthermore, when many measurements are made over space and time, expectations of the spatial correlation and temporal invariance of thermal diffusivity are valuable for validation of measurements. The explicit form also makes it somewhat faster and easier to calculate estimates from a large number of sensors or multiple positions along one sensor. Finally, the closed-form explicit solution allows for direct calculation of propagation of uncertainties in error measurements and parameter estimates, providing insight about error expectations for sensors placed at different depths in different environments as a function of surface temperature variation amplitudes. The improvements are expected to increase the utility of temperature measurement methods for studying groundwater-surface water interactions across space and time scales. We discuss the theoretical implications of the new solutions supported by examples with data for illustration and validation.