



On the action of Heisenberg's uncertainty principle in discrete linear methods for calculating the components of the deflection of the vertical

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The method of discrete linear transformations that can be implemented through the algorithms of the Standard Fourier Transform (SFT), Short-Time Fourier Transform (STFT) or Wavelet transform (WT) is effective for calculating the components of the deflection of the vertical from discrete values of gravity anomaly.

The SFT due to the action of Heisenberg's uncertainty principle indicates weak spatial localization that manifests in the following: firstly, it is necessary to know the initial digital signal on the complete number line (in case of one-dimensional transform) or in the whole two-dimensional space (if a two-dimensional transform is performed) in order to find the SFT. Secondly, the localization and values of the "peaks" of the initial function cannot be derived from its Fourier transform as the coefficients of the Fourier transform are formed by taking into account all the values of the initial function. Thus, the SFT gives the global information on all frequencies available in the digital signal throughout the whole time period.

To overcome this peculiarity it is necessary to localize the signal in time and apply the Fourier transform only to a small portion of the signal; the STFT that differs from the SFT only by the presence of an additional factor (window) is used for this purpose.

A narrow enough window is chosen to localize the signal in time and, according to Heisenberg's uncertainty principle, it results in have significant enough uncertainty in frequency. If one chooses a wide enough window it, according to the same principle, will increase time uncertainty. Thus, if the signal is narrowly localized in time its spectrum, on the contrary, is spread on the complete axis of frequencies, and vice versa.

The STFT makes it possible to improve spatial localization, that is, it allows one to define the presence of any frequency in the signal and the interval of its presence. However, owing to Heisenberg's uncertainty principle, it is impossible to tell precisely, what frequency is present in the signal at the current moment of time: it is possible to speak only about the range of frequencies. Besides, it is impossible to specify precisely the time moment of the presence of this or that frequency: it is possible to speak only about the time frame.

It is this feature that imposes major constrains on the applicability of the STFT.

In spite of the fact that the problems of resolution in time and frequency result from a physical phenomenon (Heisenberg's uncertainty principle) and exist independent of the transform applied, there is a possibility to analyze any signal, using the alternative approach – the multiresolutional analysis (MRA). The wavelet-transform is one of the methods for making a MRA-type analysis. Thanks to it, low frequencies can be shown in a more detailed form with respect to time, and high ones – with respect to frequency.

The paper presents the results of calculating of the components of the deflection of the vertical, done by the SFT, STFT and WT. The results are presented in the form of 3-d models that visually show the action of Heisenberg's uncertainty principle in the specified algorithms. The research conducted allows us to recommend the application of wavelet-transform to calculate of the components of the deflection of the vertical in the near-field zone.

Keywords: Standard Fourier Transform, Short-Time Fourier Transform, Wavelet Transform, Heisenberg's uncertainty principle.