



## **An unconditionally numerically stable super-implicit ice-sheet model incorporating membrane stresses**

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While numerical solutions of linear time-dependent equations can generally be rendered unconditionally stable by using implicit marching schemes, this is not generally the case for non-linear equations. In the case of ice-sheet models using the SIA mechanics, time-step limits required by stability considerations are generally around one year for grid sizes of a few kilometres for semi-implicit schemes. Hindmarsh (2001) showed that super-implicit schemes, with implicit weighting factors greater than or equal to  $n/2$ , where  $n$  is the Glen index, are unconditionally stable.

Hindmarsh (2009) used a related super-implicit scheme to solve the evolution equations for an ice-sheet with higher-order (membrane stress approximation) mechanics. This scheme is described here. It requires a particular form of the continuity equation, which simplifies to the usual SIA form for small membrane stress contributions. Again the scheme is unconditionally stable provided the non-linear elliptic membrane stress equations converge. The implicit weighting factor must now be  $n$  or more. Time-steps of 100a are practicable for small grid sizes.

Super-implicit schemes are unusual, but provide approximations consistent with the corresponding partial differential equations as the time-step goes to zero. They are generally under-damped, but the resulting numerical oscillations are not large, and can be reduced by reducing the time-step. The paper also discusses the use of super-implicit schemes in the context of enhanced modelling of grounding line areas, either by locally high-order spatial schemes, or by use of flux parameterisations. The ability of these computationally efficient methods to solve more general ice-sheet configurations is assessed.

Hindmarsh, R.C.A. (2001), "Notes on basic glaciological computational methods and algorithms", in Straughan, B., R. Greve, H. Ehretraut and Y. Wang, (eds.), "Continuum Mechanics and Applications in Geophysics and the Environment", p. 222-249. Springer-Verlag, Berlin Heidelberg

Hindmarsh, R.C.A., (2009), "Consistent Generation of Ice-Streams via Thermo-Viscous Instabilities Modulated by Membrane Stresses", *Geophys. Res. Lett.*, 36, L06502, doi:10.1029/2008GL036877