



# Application of Model Constrained Variational Data Assimilation Scheme on WRFVAR

Xudong Liang (1), Xiang-Yu Huang (2), Xin Zhang (2), and Jue Mei (3)

(1) Institute of Urban Meteorology, CMA, Beijing, China (xdliang@ium.cn), (2) National Center for Atmospheric Research, Boulder, Colorado, USA, (3) Meteorological Center of Air Traffic Management Bureau of East China, Shanghai, China

## 1. Introduction

The 3DVar and 4DVar has come a long way since its beginning in the late 1980s and early 1990s. In variational data assimilation scheme, usually, the analysis fields are not in balance because of poorly defined background error covariance, insufficient observations, imperfect model, and observation error. Some researches have been focused on using weak constraints to reduce the dynamic imbalance between model variables based on the idea that unbalanced initial conditions often generate high-frequency oscillations with amplitude larger than those observed in nature. One of the approaches is digital-filter initialization (DFI) proposed by Lynch and Huang (1992). Another approach is to use some physical constraint such as temporal and spatial smoothness penalty functions. Liang et al. (2007) proposed a model constrained 3DVar (MC-3DVar) technique to apply the full physics and dynamics of numerical model as constraints in 3DVar. The MC-3DVar can dramatically reduce the high frequency oscillations in the analyses fields.

In this study, the MC-3DVar technique is extended to 4DVar scheme and implemented in WRF-Var system.

## 2. The MC-4DVar technique

In MC-3DVar (Liang et al. 2007), the analysis field of the initial condition is obtained to minimize a cost function as

$$J_3 = [x - x_b]^T B^{-1} [x - x_b] + [H(x) - y]^T O^{-1} [H(x) - y] + \sigma \left[ \frac{\Delta x}{\Delta t} \right]^T R^{-1} \left[ \frac{\Delta x}{\Delta t} \right] \quad (2.1),$$

where  $x$  the analysis field,  $x_b$  the background,  $B$  the background error covariance,  $O$  the observation error covariance,  $R$  the time tendency error covariance of the model variables,  $y$  the observation,  $H$  the observation operator and  $T$  the transfer operator,  $\sigma$  is a given positive weighting.

In 4D-Var, the cost function is defined in a time window as

$$J_4 = [x - x_b]^T B^{-1} [x - x_b] + \sum_i^N [H(x_i) - y_i]^T O^{-1} [H(x_i) - y_i] + \sigma \sum_j^M \left[ \frac{\Delta x_j}{\Delta t} \right]^T R^{-1} \left[ \frac{\Delta x_j}{\Delta t} \right] \quad (2.2).$$

The first term is as same as that in 3DVar. The second term is for distance between analysis and observations at time  $i = 1, 2 \dots N$ . The third term is the penalty defined at time  $j = 1, 2 \dots M$ .

The penalty term

$$J_p = \sigma \sum_{j=1}^M \left[ \frac{\Delta x_j}{\Delta t} \right]^T R^{-1} \left[ \frac{\Delta x_j}{\Delta t} \right] \quad (1)$$

can be calculated by

$$J_p = \sigma \sum_{j=1}^M \left[ \frac{x_j - x_{j-1}}{t_j - t_{j-1}} \right]^T R^{-1} \left[ \frac{x_j - x_{j-1}}{t_j - t_{j-1}} \right] \quad (2.4).$$

Given

$$x(t_j) = M_{j-1 \rightarrow j} x_{j-1} \quad (2)$$

where  $M_{j-1 \rightarrow j}$  is the tangential linear model integrating from time  $j - 1$  to  $j$ .

## 3. One observation experiment

One observation of wind component  $v$  on 500 hPa level is assimilated in a 4Dvar scheme (4Dvar experiment) and a 4Dvar scheme with model constrains (MC experiment). The analysis increments in 0~3hr integrating in 4Dvar

and MC experiments are shown in figure 1. In the 4Dvar experiment, the high frequency oscillations are obvious compare to those in MC experiment. The RMS of the sea level pressure increment is shown in figure 2. It can be seen that the variance of the sea level pressure in 4Dvar experiment is larger than those in MC experiment and with obvious oscillations.

Figure 1 The height and wind increments during 0~3hr on 500 hPa in 4Dvar (left) and MC (right) experiments.

Figure 2 The RMS of the sea level pressure increment in 4Dvar experiment (blue line) and MC experiment (red line)