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Towards a General Theory of Extremes for Observables of Chaotic Dynamical Systems

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In this paper we derive by direct calculation the statistical properties of extremes of general observables for chaotic system, taking as mathematical framework that given by Axiom A dynamical systems possessing an invariant SRB measure. We prove that the extremes of so-called physical observables are distributed according to the classical Generalised Pareto Distribution (GPD) and derive explicit expressions for the scaling parameter σ and the shape parameter ξ . In particular, we derive that ξ has a universal expression which does not depend on the chosen observables, with $\xi = -1/(d_s + d_u/2 + d_n/2)$, where d_s, d_u, and d_n are the partial dimensions of the attractor of the set in the stable, unstable, and neutral directions, respectively. The shape parameter is negative and is close to zero when high-dimensional systems are considered. This result agrees with what derived recently using the Generalised Extreme Value approach. Combining the results obtained using such physical observable and the properties of the extremes of distance observables considered in previous papers, where a direct link is found between the ξ parameter and the Kaplan-Yorke dimension of the attractor, it is possible to derive estimates of the partial dimensions of the attractor along the stable and the unstable directions of the flow. Moreover, by rewriting the expression of the shape parameter as suitable combination of moments of the considered observable and by using the Ruelle response theory, we provide a general framework for describing the sensitivity of the shape parameter with respect to ϵ-perturbations to the flow. The possibility of defining the linear response for ξ with respect to ϵ provides mathematical arguments for suggesting that the Kaplan-Yorke dimension of the attractor changes with a certain degree of smoothness when small external perturbations are introduced, as typically found in numerical experiments with intermediate to high-dimensional chaotic systems.