On generic obstructions to recovering correct statistics from climate simulations: Homogenization for deterministic maps and multiplicative noise

Georg Gottwald (1) and Ian Melbourne (2)

(1) University of Sydney, School of Mathematics & Statistics, Sydney, Australia (georg.gottwald@sydney.edu.au), (2) University of Warwick, Institute of Mathematics, UK

Whereas diffusion limits of stochastic multi-scale systems have a long and successful history, the case of constructing stochastic parametrizations of chaotic deterministic systems has been much less studied. We present rigorous results of convergence of a chaotic slow-fast system to a stochastic differential equation with multiplicative noise. Furthermore we present rigorous results for chaotic slow-fast maps, occurring as numerical discretizations of continuous time systems.

This raises the issue of how to interpret certain stochastic integrals; surprisingly the resulting integrals of the stochastic limit system are generically neither of Stratonovich nor of Ito type in the case of maps. It is shown that the limit system of a numerical discretisation is different to the associated continuous time system. This has important consequences when interpreting the statistics of long time simulations of multi-scale systems - they may be very different to the one of the original continuous time system which we set out to study.