



A regularization of the carbon cycle data-fusion problem

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Improving our understanding of the carbon cycle is an important component of modelling climate and the Earth system, and a variety of data assimilation techniques have been used to combine process models with different types of observational data.

Here, we carry out a careful mathematical analysis on a simple, yet generic, version of the carbon allocation inverse problem. At the heart of a Bayesian approach to data-model fusion is the following problem: given a generalized observation operator H , and observations y , determine the model state x that minimizes $|Hx - y|$ in a given norm. Such a problem is well-posed if a unique solution $x = H^{-1}y$ exists, and if the inverse of H is continuous. However, in discrete models such a problem can be ill-conditioned, and hence ill-posed, when the singular values of H decay to zero.

Our analysis is carried out on the evergreen version of the Data Assimilation-Linked Ecosystem model (DALEC EV). DALEC EV depicts a forest ecosystem as a set of five carbon pools: the gross primary production (GPP) is calculated at a daily time step as a function of the foliar carbon and meteorological drivers, following a mass conservation principle the GPP is then entirely allocated to carbon pools and respiration via fluxes. While this model is very simple, it represents the basic processes simulated by more sophisticated models of the carbon cycle and the low dimension of the state variable (five carbon pools and eleven parameters) allows direct solution using otherwise hopeless methods. Using synthetic observations of net ecosystem exchange (NEE), defined as the difference between GPP and respirations, we study the conditioning of the inverse problem. We found that the generalized observation operator is ill-conditioned and we study the impact of various regularization techniques: generalized Tikhonov regularization, total least square etc. Finally we use the formalism of control theory to apply model reduction techniques to the regularization problem.