



A Modal Decomposition of the Rotating Shallow Water Equations

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The dynamics of the atmosphere and oceans are complicated because of the vast range of length and time scales involved. Understanding how energy cascades from the large to small scales is an outstanding problem in the field and of great interest. In any attempt to do this it is always necessary to specify the physical structure of the basis functions. Perhaps the most popular choice are Fourier modes, which are desirable because they 1) can form a complete basis; 2) are well understood because of the richness of Fourier analysis; and 3) are a basis for high-order spectral methods. This is a convenient choice but numerous other possibilities exist, such as polynomials and wavelets. All of these choices are generic in that they do not arise from the underlying physics of the waves and can usually be applied to virtually any problem. The motivation for this work stems from the idea that a better choice for basis functions should be dictated by the model equations.

One relatively simple model that has often been used to look at energy transfers between different length and time scales is the Rotating Shallow Water model (RSW). It is restrictive in that it only describes homogeneous fluids, however, because it can contain both fast gravity and slow Rossby waves it is a useful paradigm to study energy transfers between waves with vastly different scales. The pioneering work of Leith (1980) investigated the decomposition of the RSW into its linear modes and subsequently others have built on this to understand the modal structure of stratified flows. In these works the emphasis has been on f-plane and therefore the slow component was a vortical mode that does not propagate.

In his original paper Leith points out that it would be interesting to extend his methodology to a beta-plane and in this talk we present results from our preliminary work to do just that. This is done numerically using spectral methods to find the most accurate solutions possible for a given number of degrees of freedom. First, we determine the structure of the linear RSW modes on a beta-plane in meridional channel. In the continuous limit these waves form a complete basis and are a natural set of basis functions to study in this model and have extensions in other models. Second, we present results from a series of numerical experiments of both freely-evolving and forced flows, to address how energy is transferred between the linear waves. This will consist of wave-wave interactions as well as geostrophic turbulent flows.